Semantics for Means-End Ascriptions

Jesse Hughes
Section of Philosophy and Ethics of Technology
Technical University of Eindhoven
JHughes@tm.tue.nl

1. The problem

1.1 Formal semantics for technical functions

This research forms part of the NWO *Norms in Knowledge* project. One of the main aims of this project is the formalization of artifactual functions.

Artifacts are typically intended for certain uses. A toaster makes toast and a lighter creates small fires. These are the primary functions of toasters and lighters, respectively.

We aim to provide a conceptual analysis of such functions via formal semantics. That is, we wish to construct a logical language in which functional ascriptions to artifacts can be expressed and a corresponding semantics which assigns meanings to such statements. We present today the initial steps to this goal.

1.2 Formal semantics for means-end ascriptions

Functional ascriptions entail related means-end ascriptions. When we say that the function of the toaster is to make toast, we are committed to the claim, "The toaster is a means to producing toast." More precisely, we are committed to the claim, "There is some way of using the toaster that will produce toast."

We believe that means-end ascriptions are very closely related to functional ascriptions. Indeed, an analysis of means-end ascriptions is an essential first step to our conceptual analysis of functions. We report today on some of the progress on this initial step. We investigate a language in which

- ends are formulas, i.e. *descriptions* of the world;
- means are transitions between possible worlds, i.e. *actions*.

2. The approach

2.1 Dynamic logic

An end is a description. A means is some way to change the world so that the end is realized. This strongly suggests that we take dynamic logic as our basic language.

Propositional dynamic logic (PDL) [4] is built from a set of atomic propositions and a set of actions. For each action a, we introduce modal operators $[a]$ and $\langle a\rangle$.

We interpret . . .	as asserting that. . .
$[a]\varphi$	doing a *will* result in φ;
$\langle a\rangle\varphi$	doing a *may* result in realizing φ.

The standard semantics for dynamic logic consists of a set of possible worlds together with a non-deterministic labeled transition system. The transition system gives the set of possible outcomes of doing a in a given world.

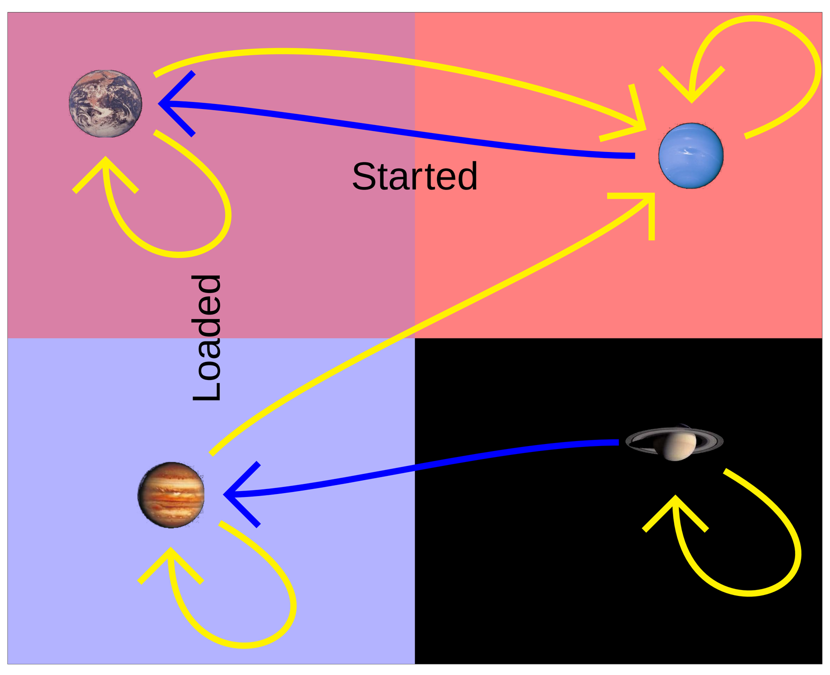


Figure 1: *A PDL model for a race started by firing a one-shot pistol, with the possibility of misfires.*

In Figure 1 we model a situation in which one has a (one-shot) starter pistol used for starting a race. The pistol may be loaded or unloaded and the race may have started or not. The yellow arrows show the effect of firing the pistol and the blue arrows the effect of loading the pistol. To make it more interesting, we include the possibility that a loaded gun misfires (this does not change the world at all).

> **Definition:** In a world w, an action a is *a (local) means to* φ iff
> 1. one can do a in w and
> 2. doing a in w ensures that φ will be realized.

The first condition is represented by the formula $\langle a\rangle\top$ (where $\top$ is the always true predicate) and the second by $[a]\varphi$.

> a is a means to φ in w $\Leftrightarrow$ $w \models \langle a\rangle\top \wedge [a]\varphi$.

Table 1 applies this definition to Figure 1.

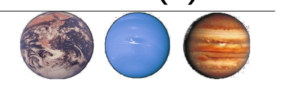
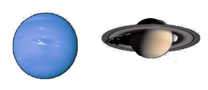
Action	is a means to. . .	in world(s). . .
fire	Started	
fire	$\neg$Started	
load	Loaded	
load; fire	Started	

Table 1: *Some means-end ascriptions for Figure 1.*

2.2 Logic of ability – a comparison

Mark Brown constructs a "logic of ability" in [1]. He represents the "can" of ability by a modal operator ☼.

> ☼φ ***means*** one *can* reliably bring about φ.

Brown's semantics are a possible world semantics involving *minimal models* (i.e. *neighborhood semantics*).

One expects a close connection between *can* assertions and *means-end* assertions, something like the following:

> One *can* bring about φ
> $\Updownarrow$
> There is a *means* to φ

In fact, if nothing like this relation holds, then we would be concerned about the appropriateness of our means-end semantics.

Fortunately, we do get the desired correspondence. There is a natural translation of our means-end language to ability, so that the can-operator essentially captures the existence of means, as one expects. This fact provides some confirmation of the sensibility of our approach.

Brown's logic offers more than just peace of mind, however. He uses minimal models to avoid some undesirable features of Kripke models. In particular, if one uses Kripke models and the strong modal operator to model ability, then one is committed to the equivalence

> Property of Kripke frames
>
> ☼$\varphi \wedge$ ☼ψ $\Leftrightarrow$ ☼$(\varphi \wedge \psi)$
>
> (one can φ *and* one can ψ) $\Leftrightarrow$ one can (φ *and* ψ)

(The weak operator has a dual property that is equally undesirable.) This equivalence conflicts with basic facts about ability. Often, we are able to bring about either of two mutually exclusive conditions:

> (one can close the door *and* one can leave it open) *but not* both at once!

The same considerations apply to means-end ascriptions. It makes sense to use minimal models for our means-end semantics instead of standard Kripke models. Nonetheless, we present the simpler Kripke models today.

3. Refinements

3.1 Global/conditional ascriptions

We have defined the term "means in a world", but this is a very narrow kind of means-end ascription. Very often, one is interested in broader assertions, such as *normally* (but not always), a is a means to φ. We summarize three different types of means-end ascriptions in Table 2.

Type	Meaning	Formula
local	in *this* world, a can yield φ	$[a]\varphi$
conditional	given ψ, a normally yields φ	$\psi \Rightarrow [a]\varphi$
global	a normally yields φ	$\top \Rightarrow [a]\varphi$

Table 2: *Types of means-end ascriptions (with* $\langle a\rangle\top$ *suppressed).*

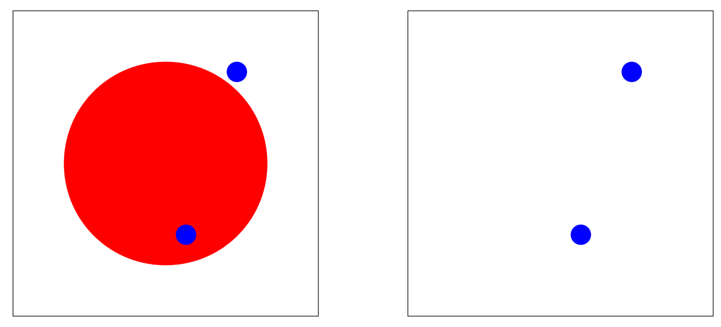
For example, one would like to say that firing the gun is a means to starting the race, *provided* that the gun is loaded. An essential feature of practical reasoning of this sort is *non-monotonicity*, better known as *the frame problem* [2]. If the gun is loaded, then firing it will start the race, but not if all of the racers happen to be hearing-impaired!

> $\mathrm{Loaded} \Rightarrow [\mathrm{fire}]\mathrm{Started}$,
> *but*
> $\mathrm{RunnersDeaf} \wedge \mathrm{Loaded} \not\Rightarrow [\mathrm{fire}]\mathrm{Started}$.

We accommodate the frame problem by interpreting "$\Rightarrow$" as a *non-monotonic conditional operator*.

3.2 Efficacy and fuzzy sets

A central feature of means (and technical functions) is efficacy: the propensity to attain one's end. Traditional semantics fail to represent efficacy. For this, one wants probabilistic features while retaining truth-functionality. We do this by using fuzzy set semantics [3, 5].

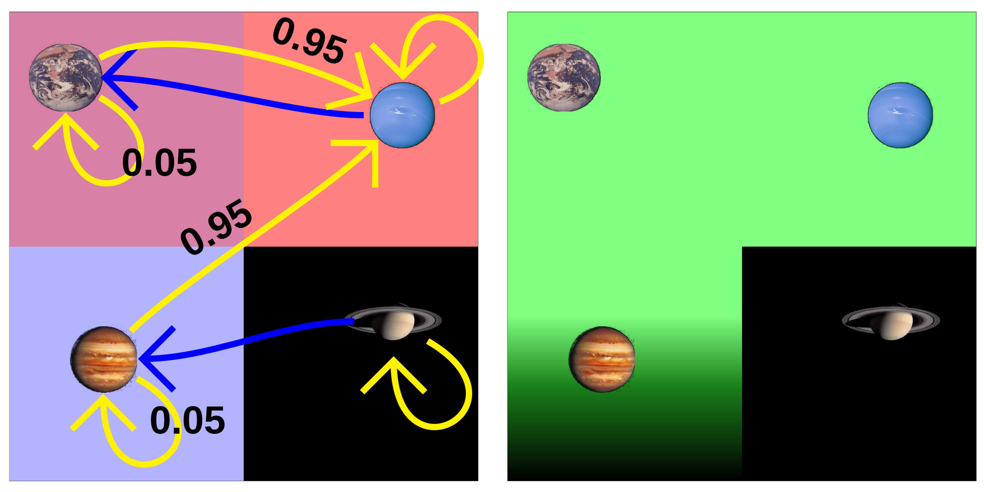


(a) (b)

Figure 2: *In classical set theory (a), an element is either in a set or not;* $x \in S$ *is either true or false. In fuzzy set theory (b), the formula* $x \in S$ *is true to some degree.*

We add probabilities to our transition systems. In our example, a misfire is less likely than otherwise and our transition structure should reflect that. Our semantics reflect uncertainty via fuzzy logic. This extends traditional fuzzy logic by adding dynamic operators and novel semantics for same.

In the end, the set of worlds satisfying a given PDL formula is a *fuzzy set*, as shown in Figure 3.



(a) (b)

Figure 3: *Fuzzy semantics for PDL: In (a), we assign probabilities to some of the transitions, resulting in the fuzzy set of worlds satisfying* $[\mathrm{fire}]\mathrm{Started}$ *illustrated in (b).*

4. Future work

4.1 A return to functions

Once we are satisfied with our basic means-end semantics, we will return to artifactual functions. This will involve parameterizing the dynamic operators with tokens and/or types, shown below.

Parameter	Operators
tokens	$\langle\mathrm{fire}(\text{this gun})\rangle$, $[\mathrm{fire}(\text{that gun})]$
types	$\langle\mathrm{fire}(\text{starter gun})\rangle$, $[\mathrm{fire}(\text{flare gun})]$

Of course, there are still many open questions regarding functions:

- How to distinguish proper from accidental function?
- How to characterize malfunction?
- Can one represent other evaluative features besides efficacy?
- And so on. . .

References

[1] Mark Brown. On the logic of ability. *Journal of Philosophical Logic*, 17:1–26, 1988.

[2] Daniel C. Dennett. Cognitive wheels: The frame problem of AI. In Margaret A. Boden, editor, *The Philosophy of Artificial Intelligence*, Oxford Readings in Philosophy, pages 147 – 170. Oxford University Press, 1990.

[3] Petr Hájek. *Metamathematics of Fuzzy Logic*, volume 4 of *Trends in Logic*. Kluwer Academic Publishers, 1998.

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[5] George J. Klir and Bo Yuan. *Fuzzy Sets and Fuzzy Logic: Theory and Applications*. Prentice Hall PTR, 1995.