Of Functions and Means: An exercise in applied logic

Jesse Hughes

Eindhoven University of Technology

February 17, 2007

Outline

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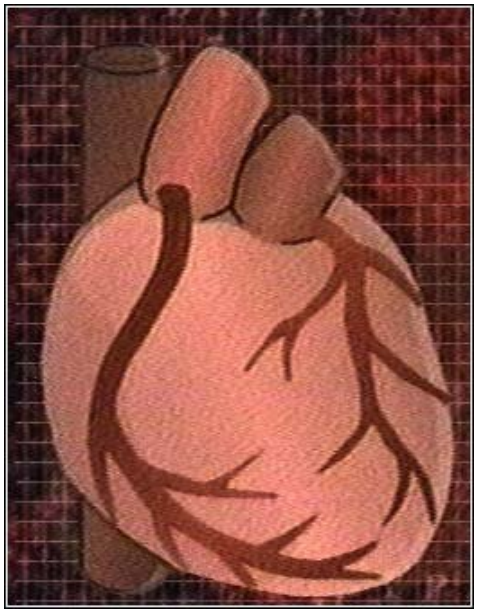
Functional ascriptions



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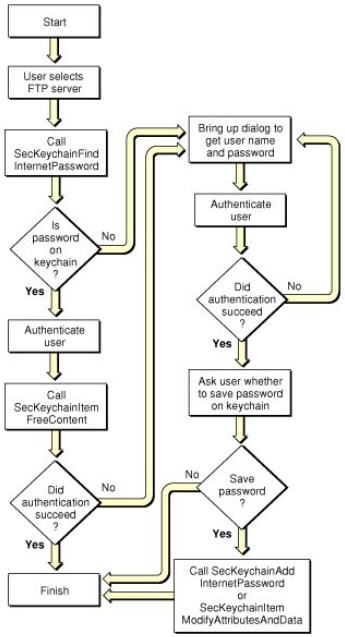

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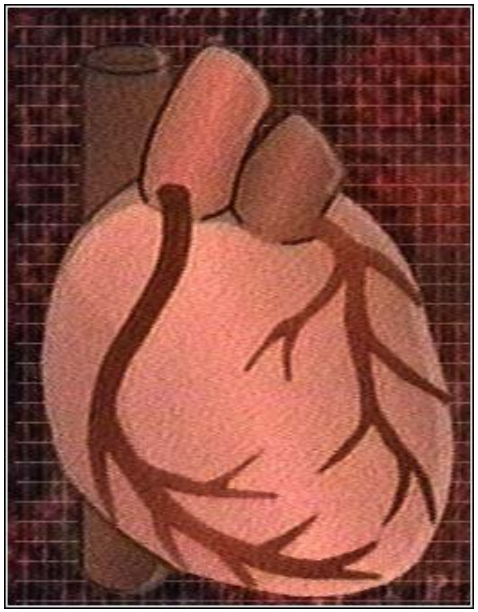

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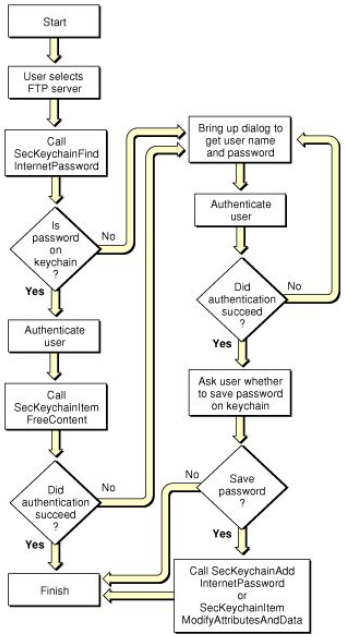

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We ascribe functions to biological stuff, artifacts, algorithms, personal roles... but is one notion of function enough?

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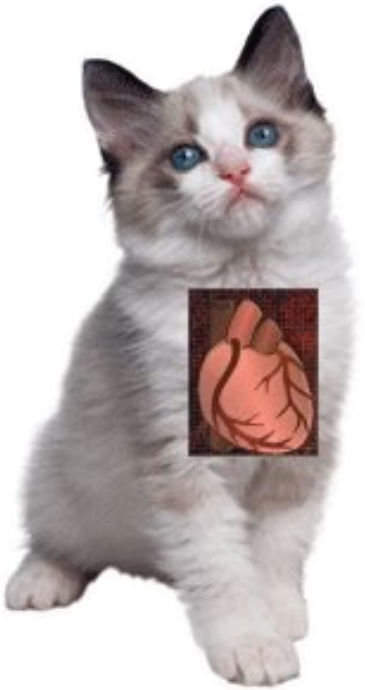

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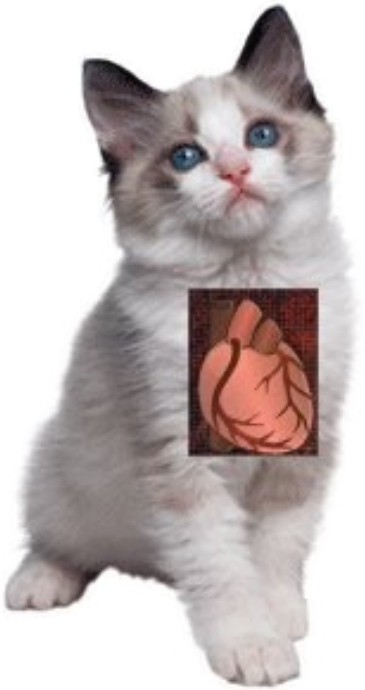

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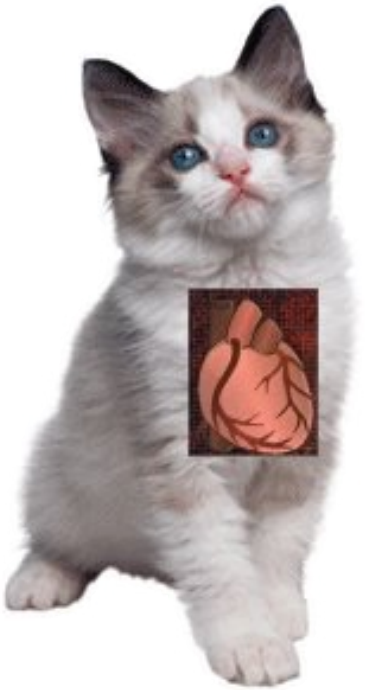

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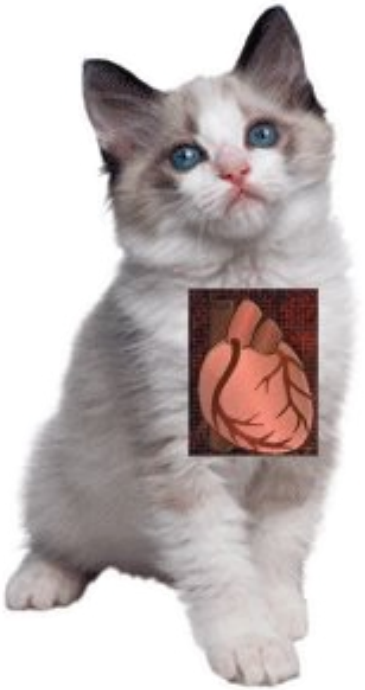
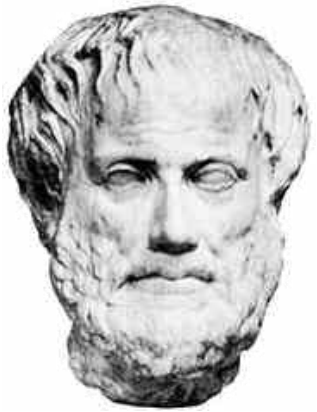
  - The remote was created because previous remotes sold well.



Functions do more than explain. . .

Functions explain stuff.

- How parts contribute to the whole.
- How things came to be as they are.



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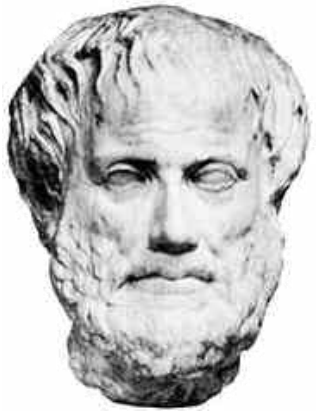

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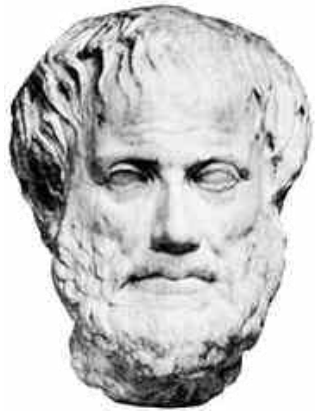

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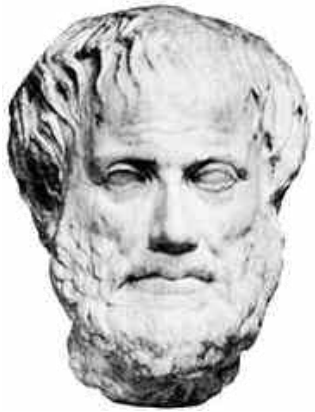

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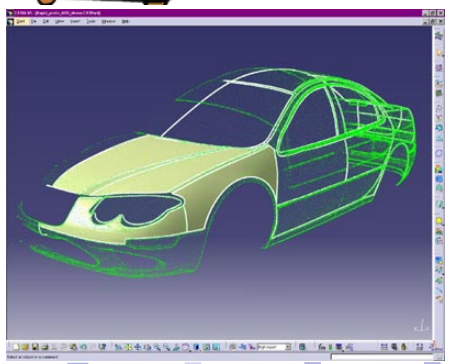

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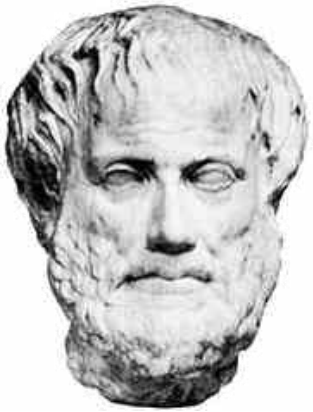

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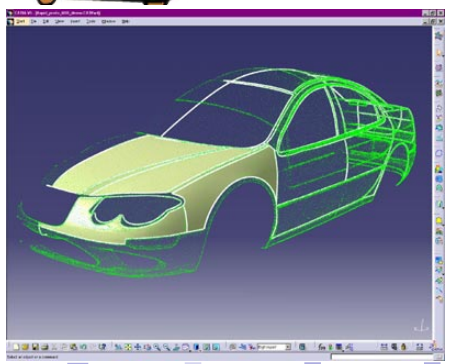
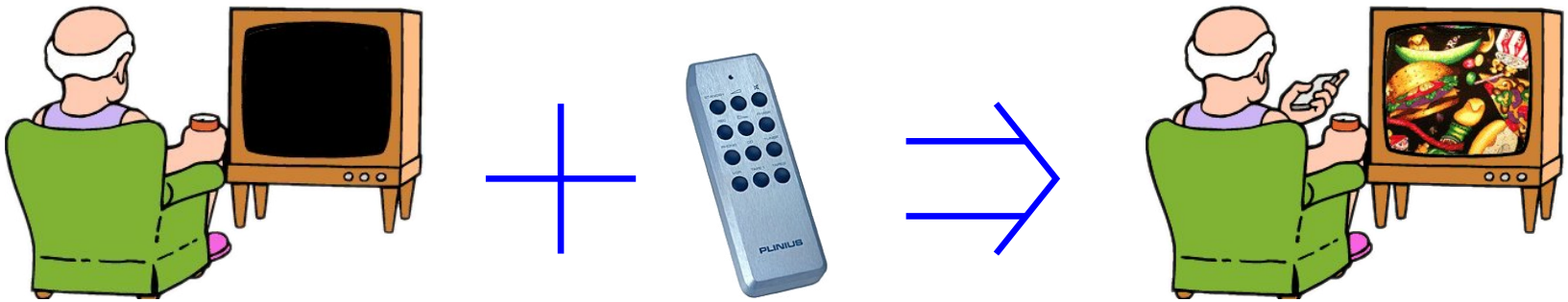
Artifact functions have practical importance.



Functions have instrumental consequences

Artifactual knowledge produces practical knowledge.



desire + function ⇒ action

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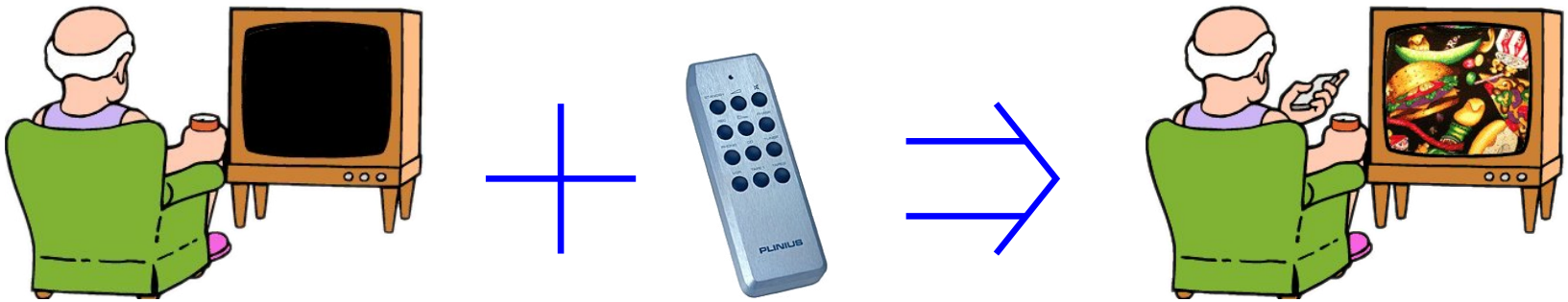

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Functions yield means-end relations!

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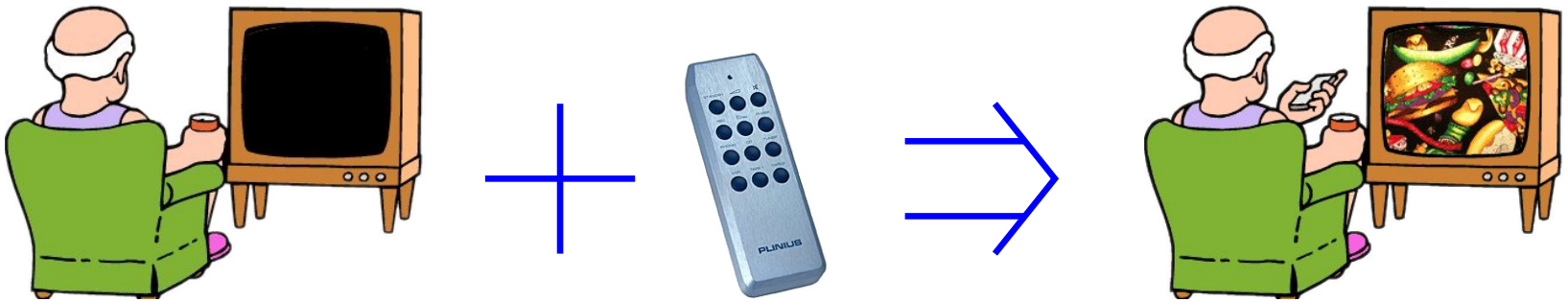

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Means-end relations can be given a formal semantics.

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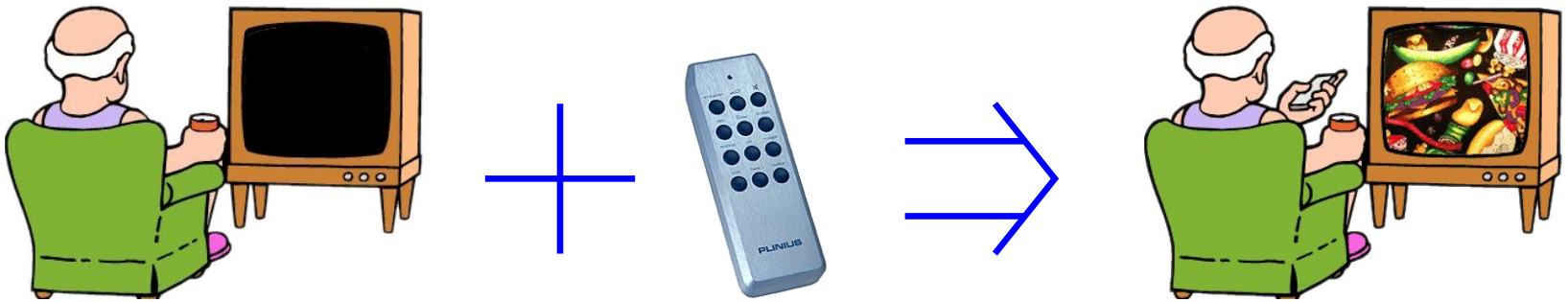

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Use this to understand certain function talk: malfunction, failure, efficacy, etc.

Outline

Conceptual starting points

- An end is a condition to be realized.

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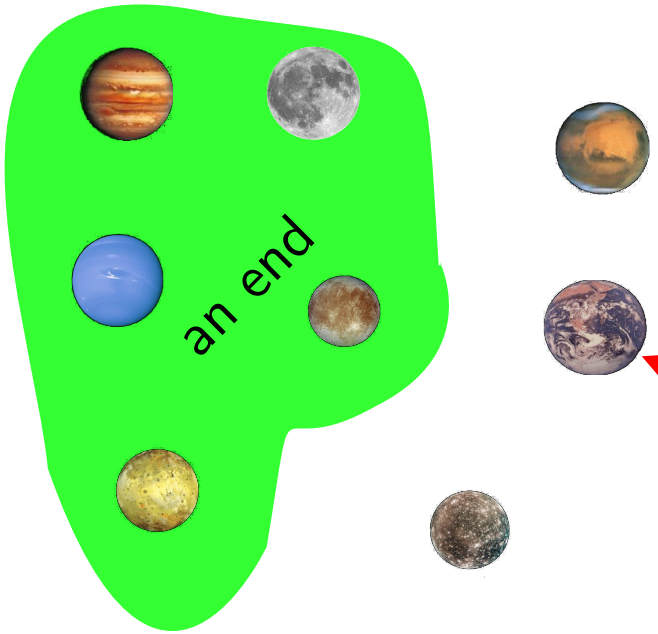

- An end is a condition to be realized.

You are here.

Think possible worlds!

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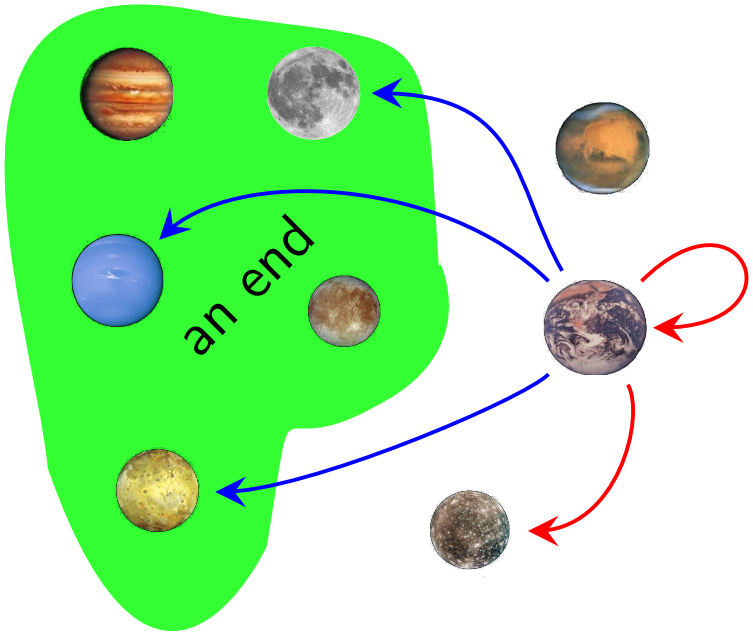

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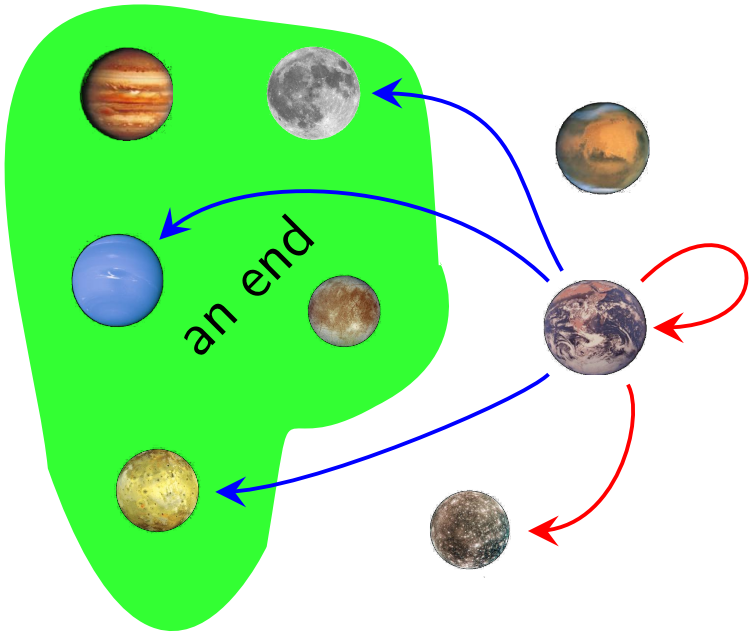

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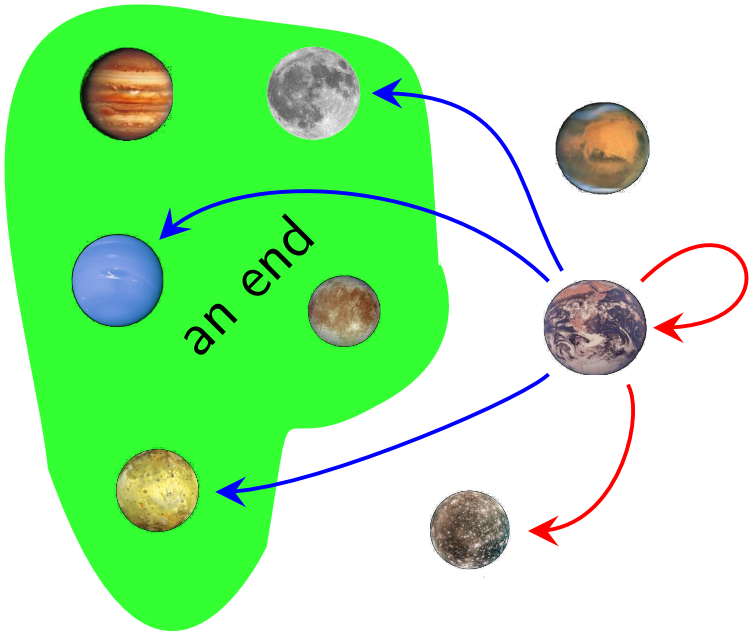

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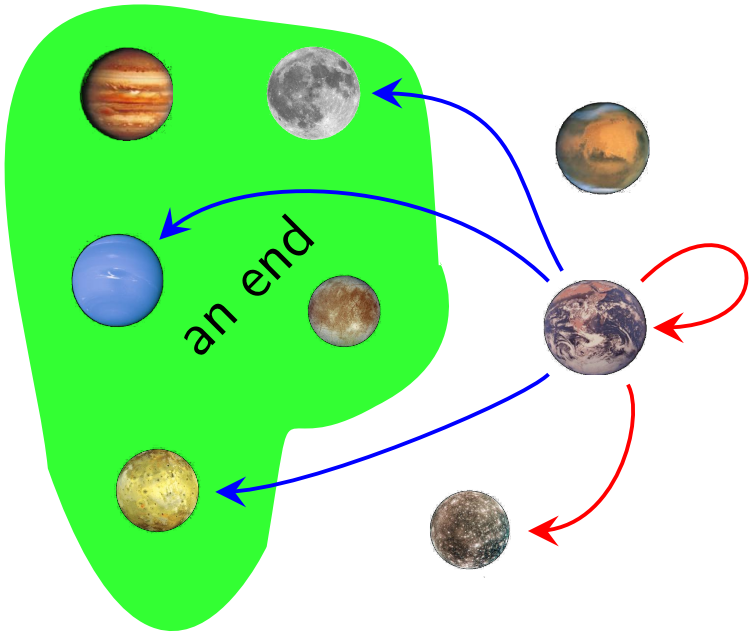

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- Propositional Dynamic Logic is a natural setting.

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- $\langle\alpha\rangle\varphi$: after doing α, φ *might* hold.

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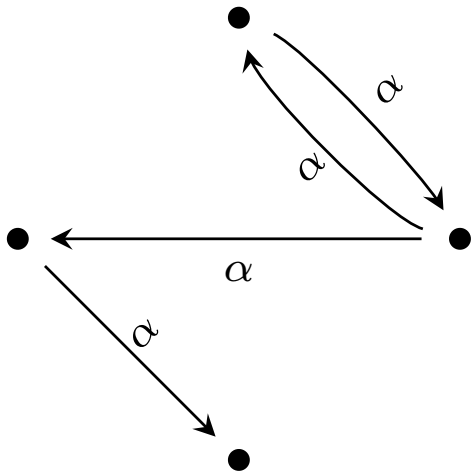

Possible world semantics with transition systems for each action α.

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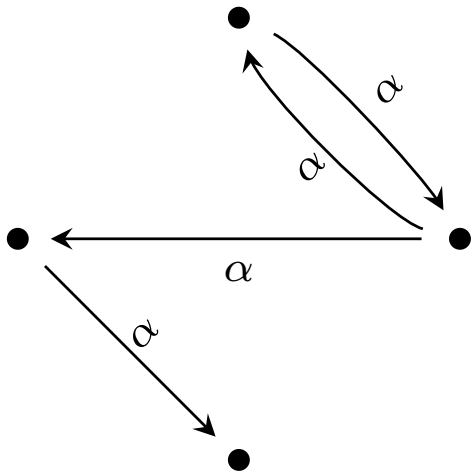

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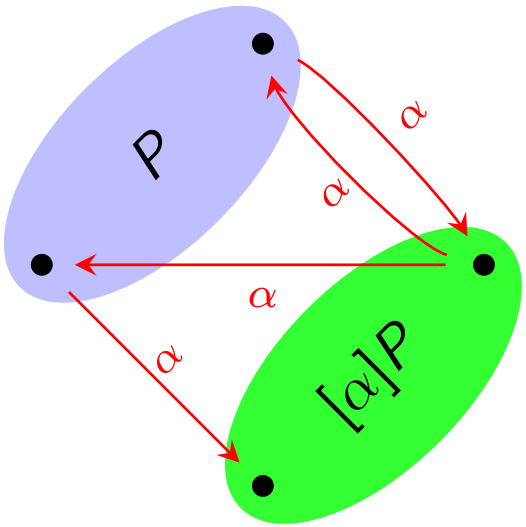

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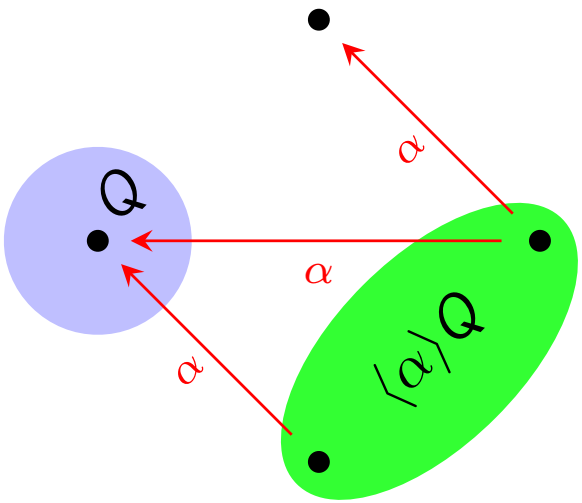

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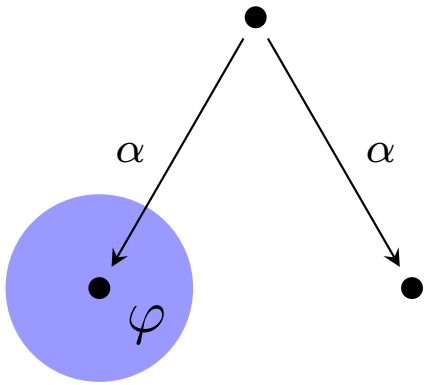

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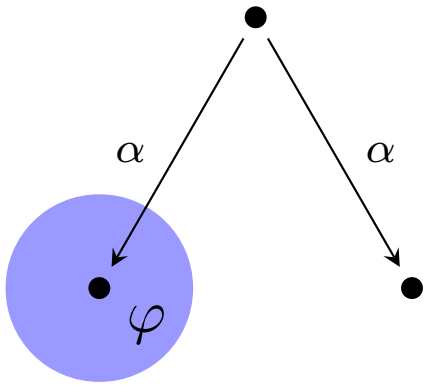

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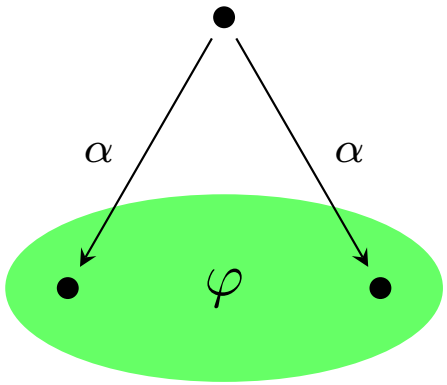

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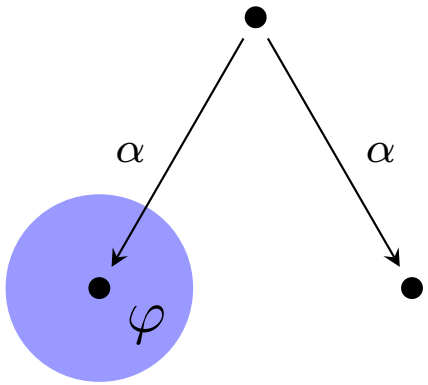

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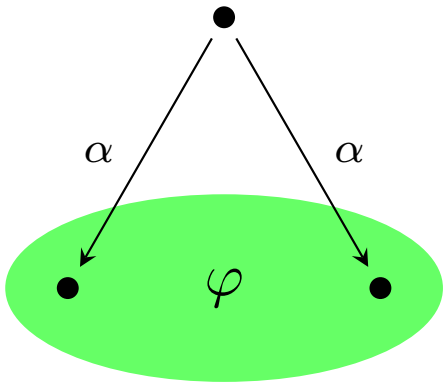

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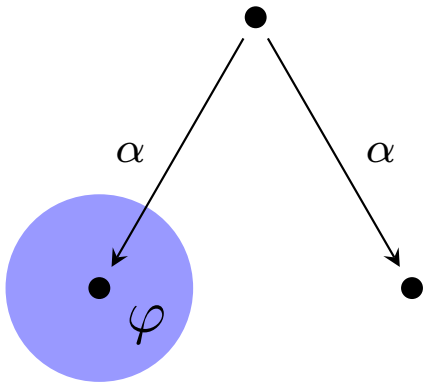

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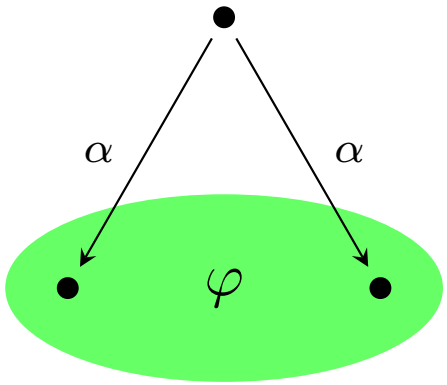

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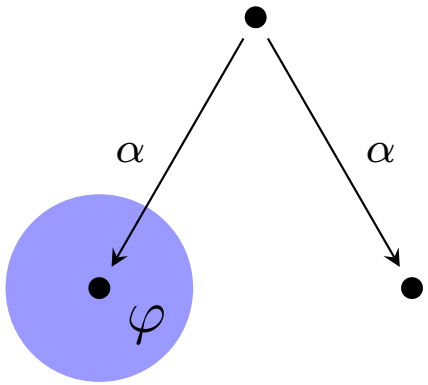

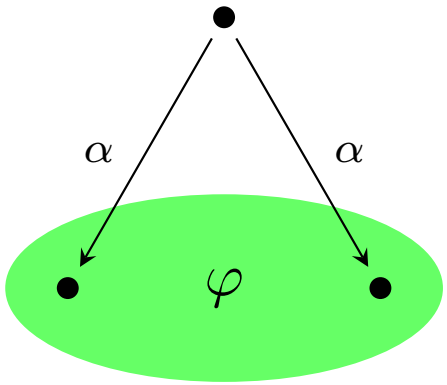

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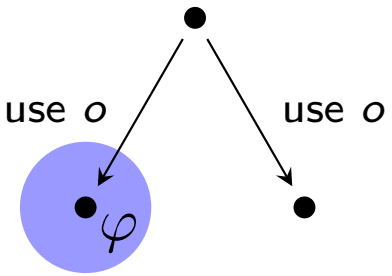
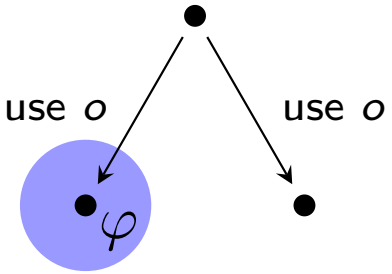
Caveat: This definition omits relevance.

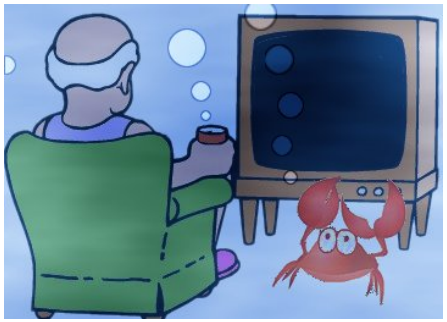
From functions to means: the proposal



If the function of o is to bring about φ, then "use o" is a weak means to φ...

From functions to means: the proposal





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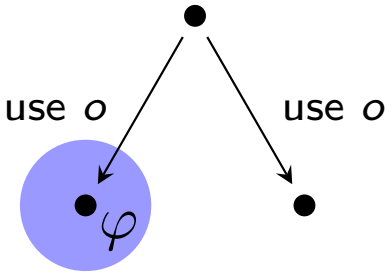

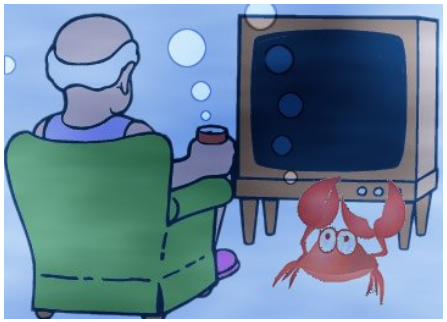

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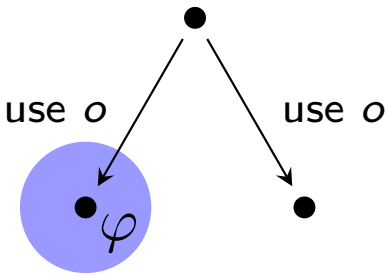

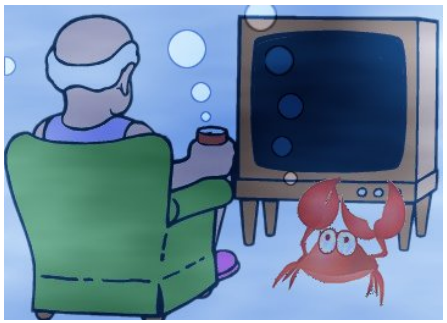

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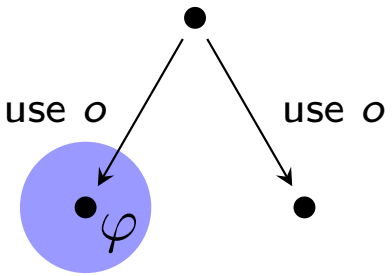

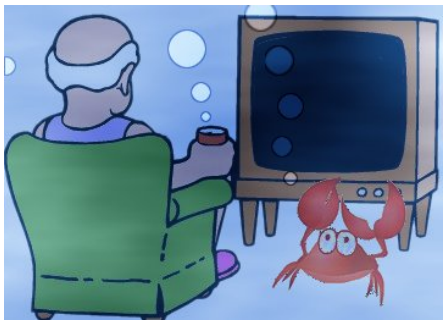

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Context matters!

Lesser known features of function ascriptions

"That button turns on the television."

No context apparent there! Just a type and an end.

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Given a functional ascription:

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This expectation is general. Success or failure comes in particular applications.

Particular uses and their models

Application: using a token $o \in T$ in a situation $c \in C$.

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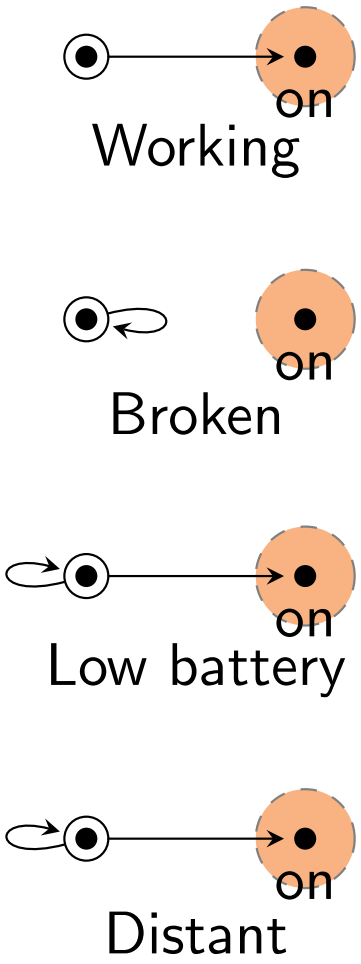

Particular uses and their models

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o and c provide parameters for α and φ.

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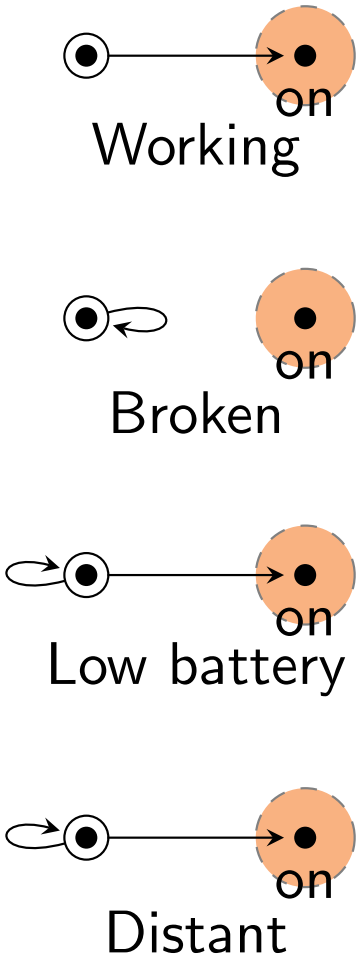
function knowledge $\Rightarrow$ practical expectations

particular uses $\Rightarrow$ success/failure



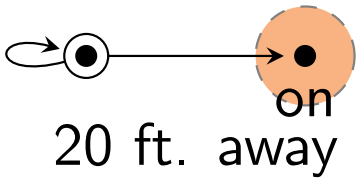
Outline

Limitations of non-determinism

But non-deterministic models only go so far.

Suppose our remote works up to 40 ft.

Limitations of non-determinism



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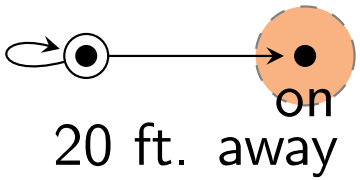

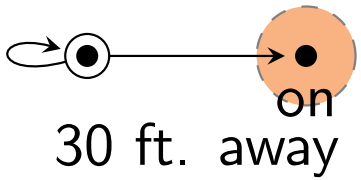

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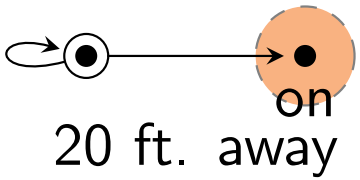

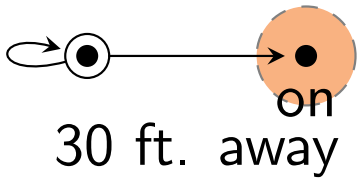

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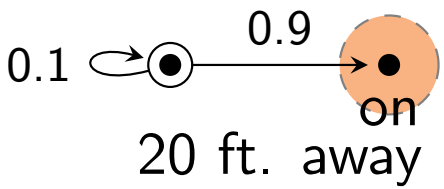
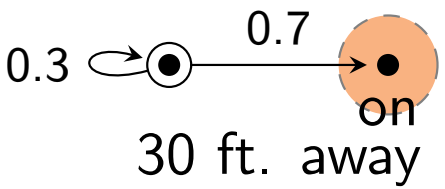
At 20 ft., the remote is more reliable than at 30 ft.

Our non-deterministic models can't distinguish these situations.

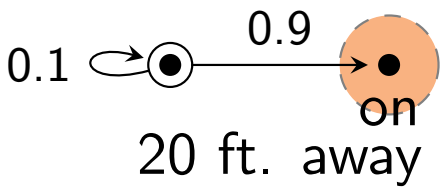
Adding probabilities to dynamic logic

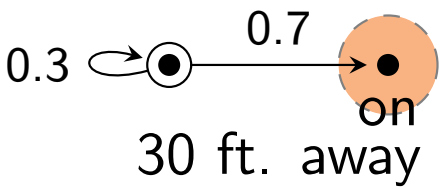




We need a probabilistic transition system.

Adding probabilities to dynamic logic





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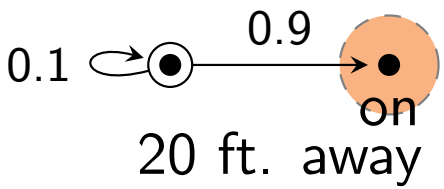

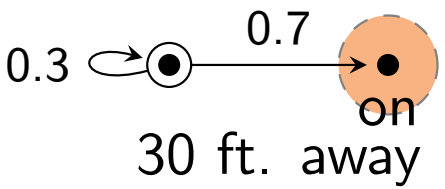

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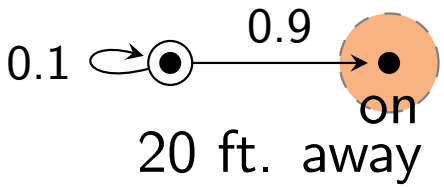

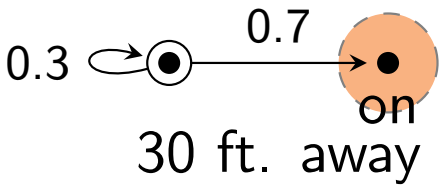

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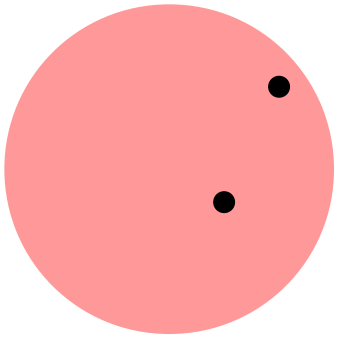
Fuzzy!

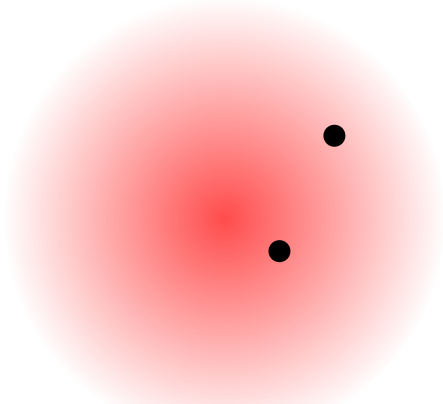
Reliability is vague.

Vague predicates can be modeled by fuzzy sets.

Fuzzy PDL



A crisp proposition



A fuzzy proposition

In fuzzy logic, a proposition may be more or less true.

Truth comes in degrees.

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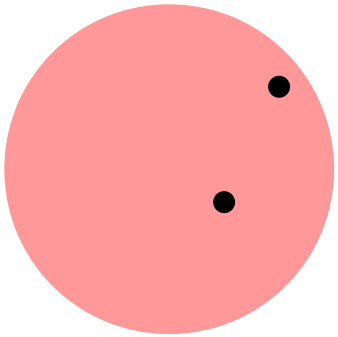

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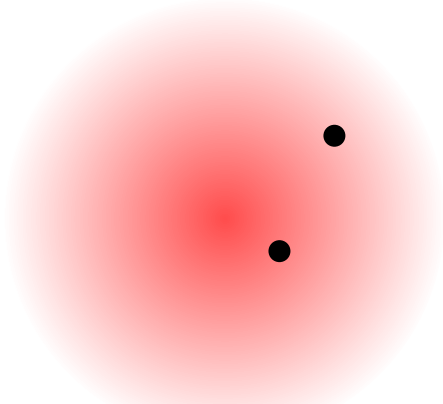

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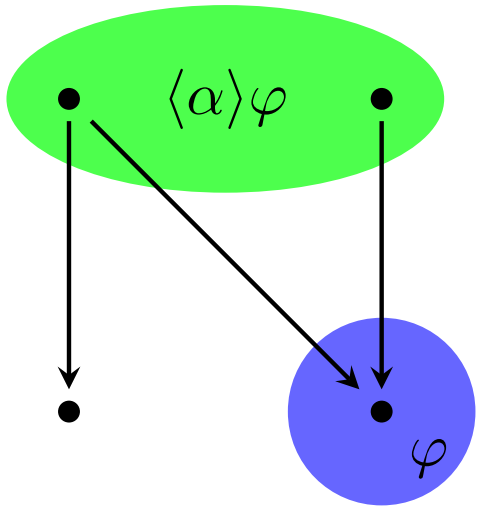

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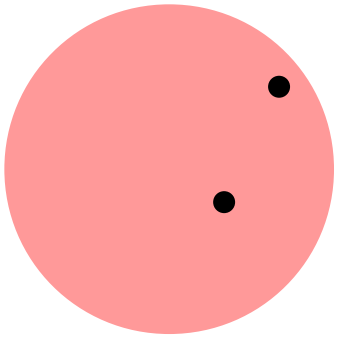

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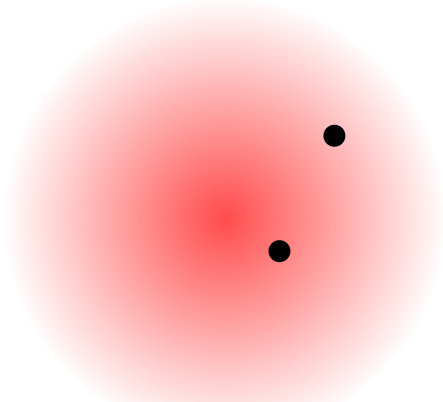

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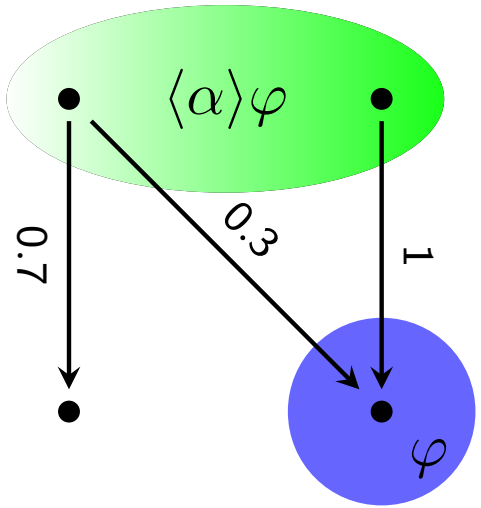

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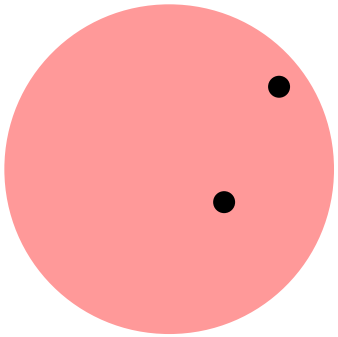

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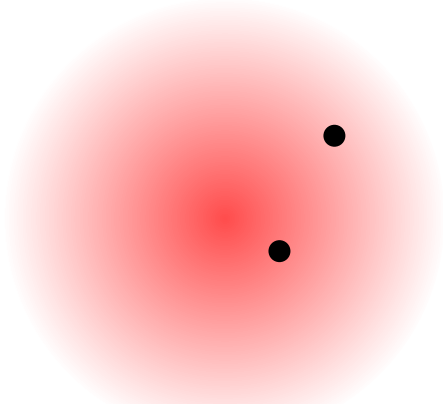

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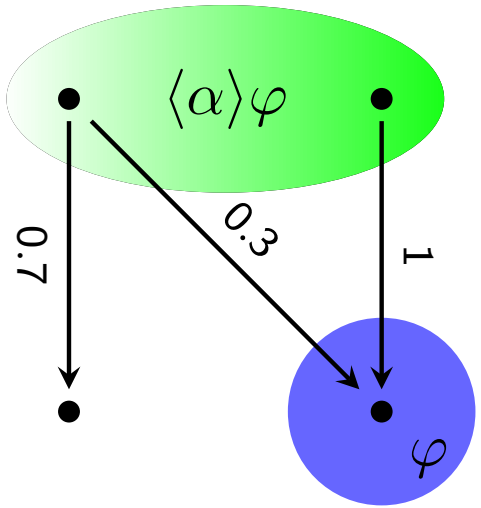

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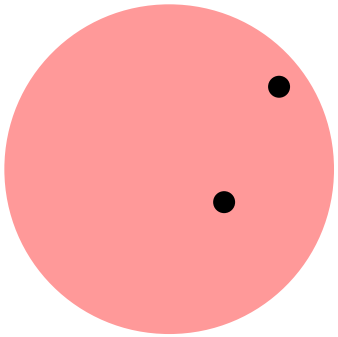

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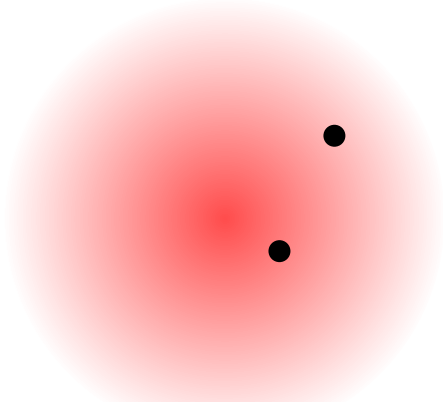

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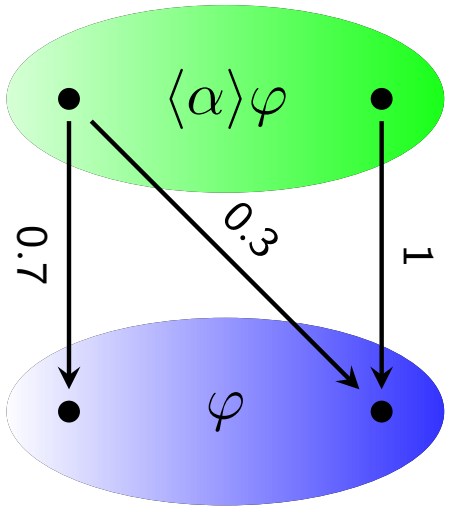

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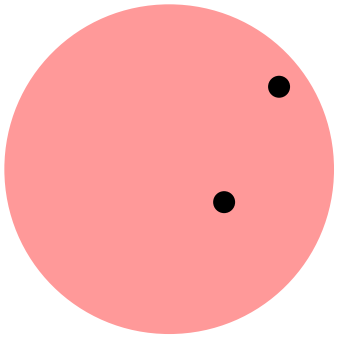

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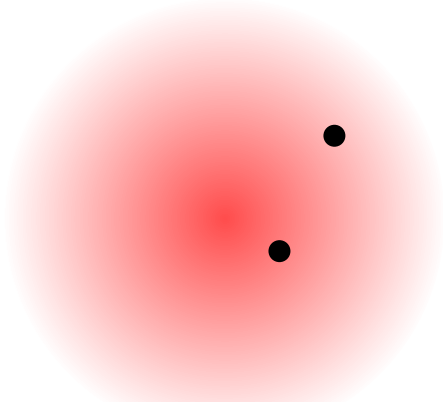

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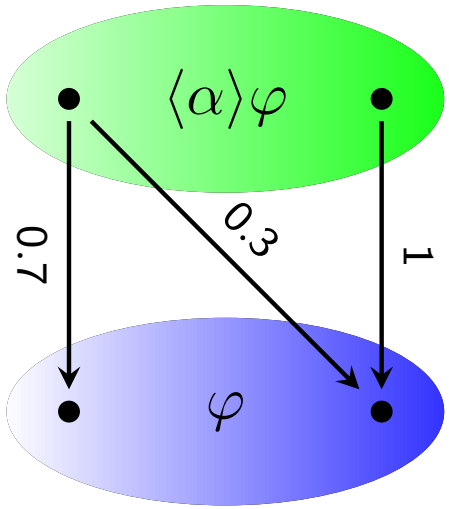

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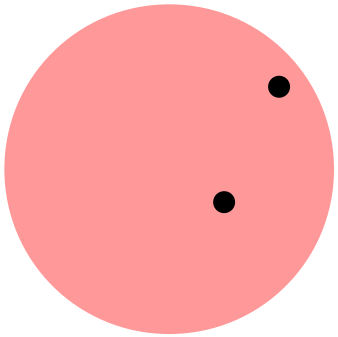

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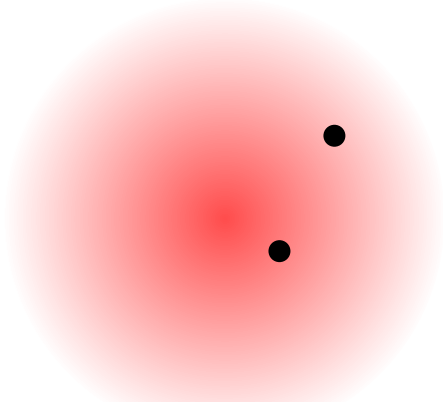

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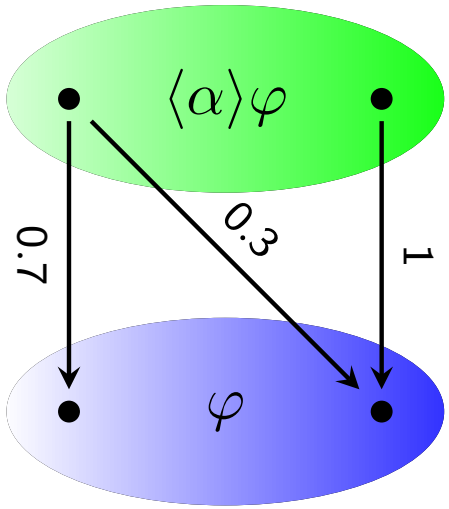

Malfunction and failure

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Common definition: A token malfunctions when it cannot do what it is supposed to do.

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A brief introduction to normality

Normal tokens are abstractions representing expectations.

- Users gain experience from use.

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A brief introduction to normality

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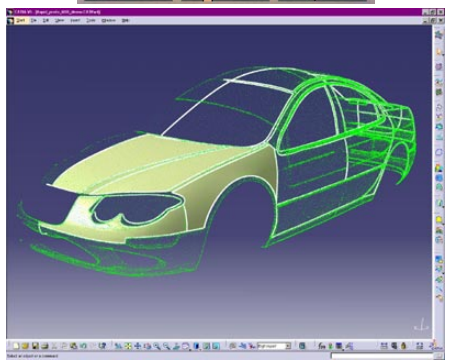

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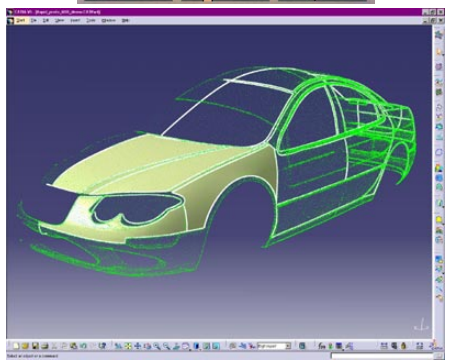

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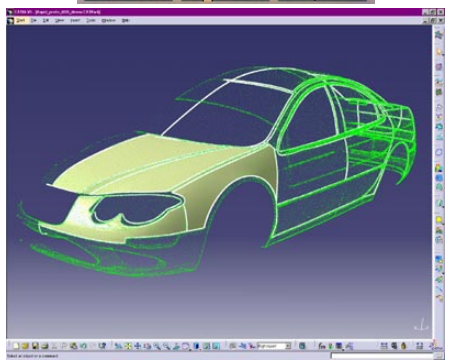
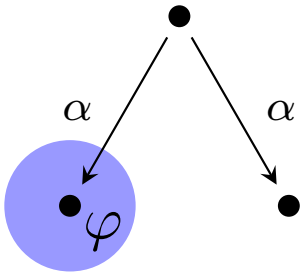
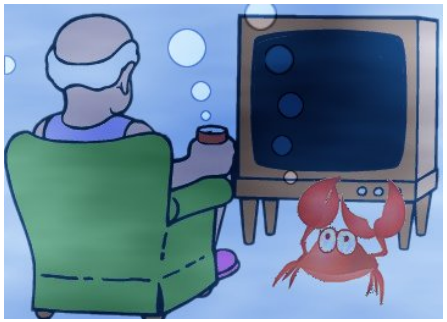


So what did we get?





- The relation between functional knowledge and practical reasoning

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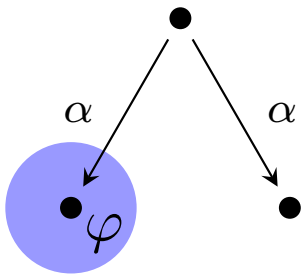

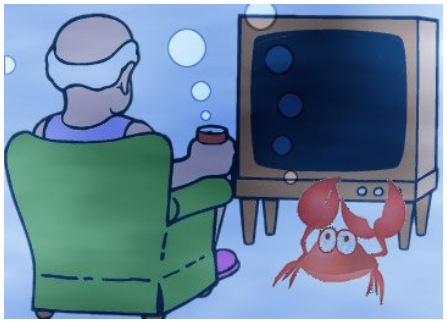

- The relation between functional knowledge and practical reasoning
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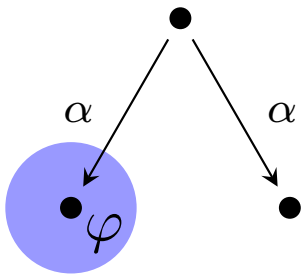

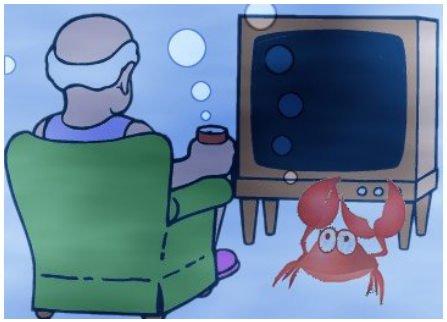

- The relation between functional knowledge and practical reasoning
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- Fuzzy PDL, giving
  - Notion of efficacy of means
  - Representation of vague ends
  - Token-token, token-type and type-type comparisons for artifacts

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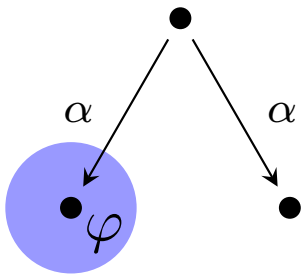

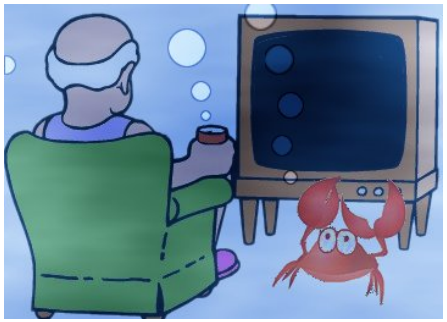

- The relation between functional knowledge and practical reasoning
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  - Notion of efficacy of means
  - Representation of vague ends
  - Token-token, token-type and type-type comparisons for artifacts
- Distinction between malfunction and failure

Thank you!