A Semantics for Means-End Ascriptions

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Outline

1. Means and ends, informally
   - Norms in Knowledge
   - Initial analysis
2. Means and ends, formally
   - Propositional Dynamic Logic
   - Brown's Logic of Ability
3. Means and ends, fuzzily
   - Why go fuzzy?
   - Fuzzy sets

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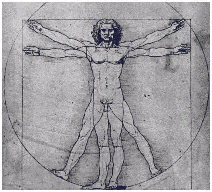
An epistemological investigation.



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- Knowledge of descriptive claims
- Knowledge of normative claims
  - Non-moral

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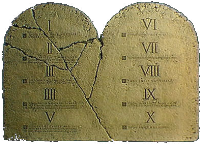

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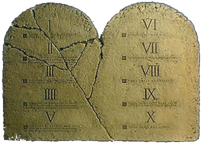

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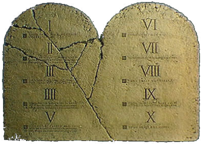

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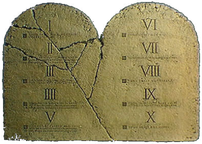

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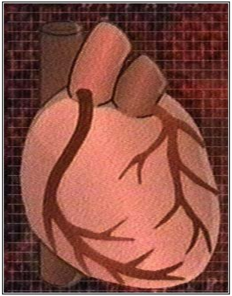

- "The function of the heart is to pump blood."
- "That switch mutes the television."
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We ascribe functions to biological stuff,

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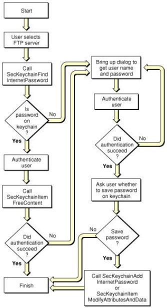

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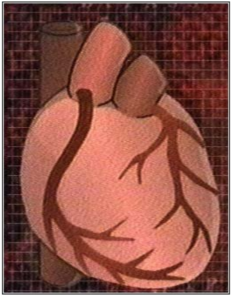

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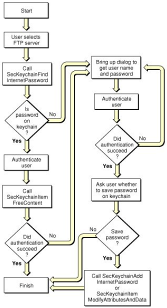

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How functions relate to means and ends

"That switch mutes the television."

$\Downarrow$

One can use the switch to mute the television.

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There is an action involving the switch that will cause the television to be muted.

- Functions imply means-end relations.
- Step one: Provide a semantics for means-end relations.

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What is an end? a mean?

- An end is some desirable condition.
- A means is a way of making the end true.
- Means change things: means are *actions*.

Some controversies.

- Ends-in-themselves?
- Objects as means?

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Possible worlds and making propositions come true

Ends are propositions we want to make true.

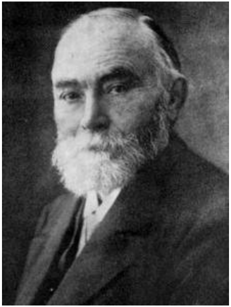
But actions don't change the meaning of propositions.

Think of a set of possible worlds.

At each time, *one* world is the actual world.

And at each world, every proposition is true or false.

Possible worlds and making propositions come true



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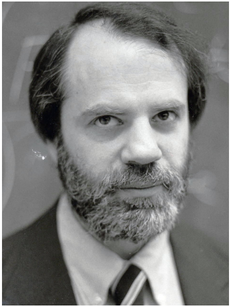

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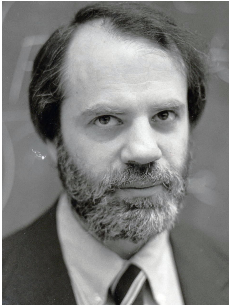

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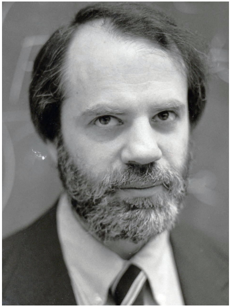

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A simple example of possible worlds

Our Universe

A set of worlds involving
a footrace and starter pistol.

- Two basic properties:
    - Footrace started?
    - Pistol loaded?

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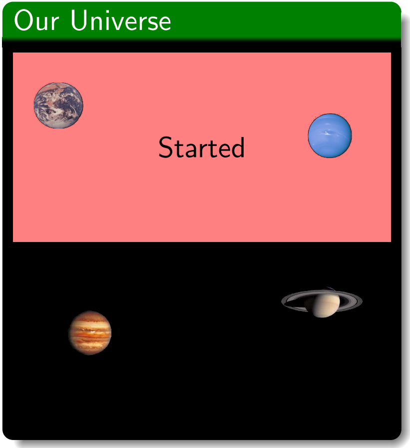

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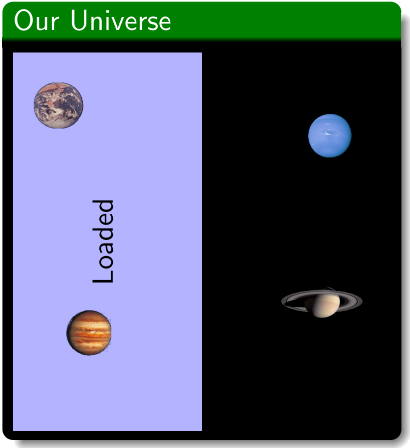

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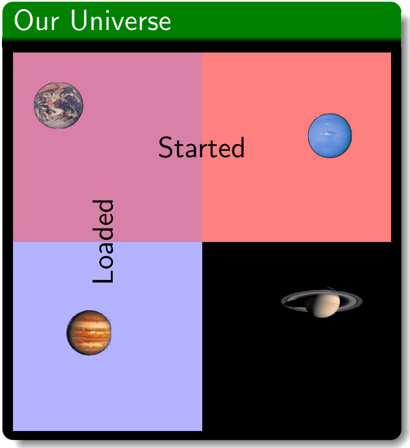

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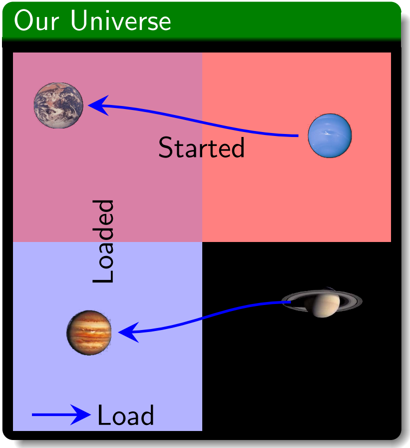

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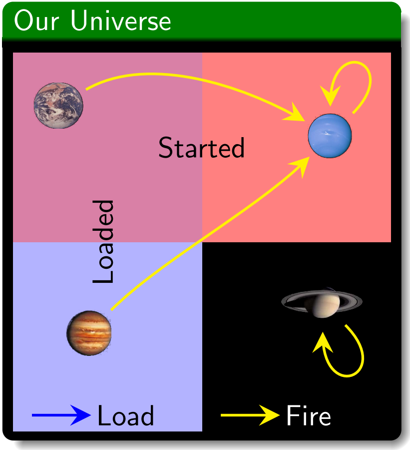

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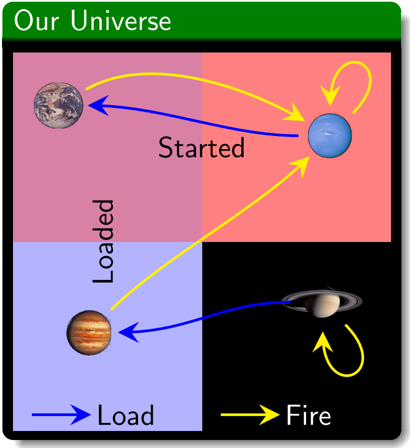

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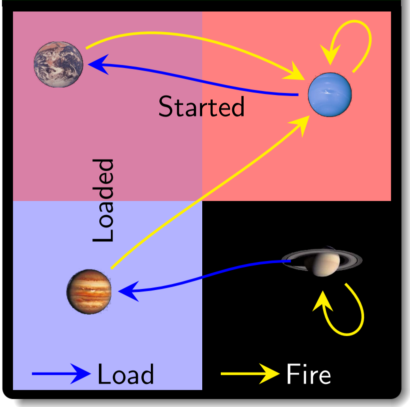

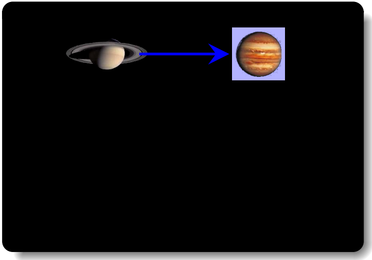

In Saturn, loading and firing is a means to starting the race.

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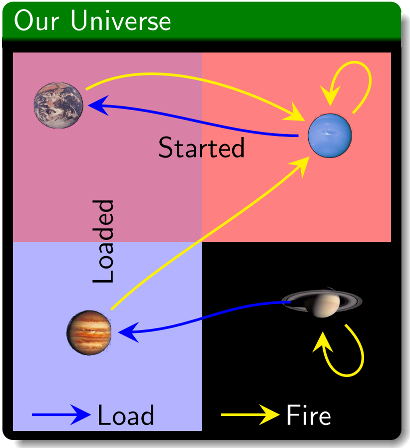

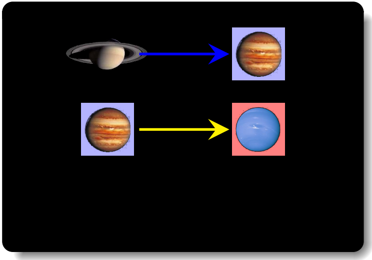

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Some PDL examples

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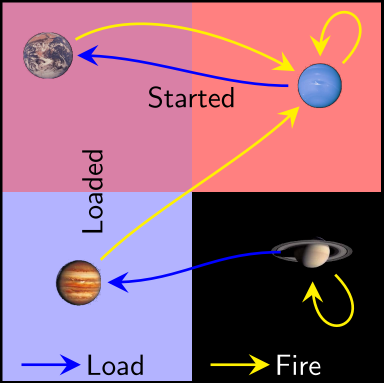

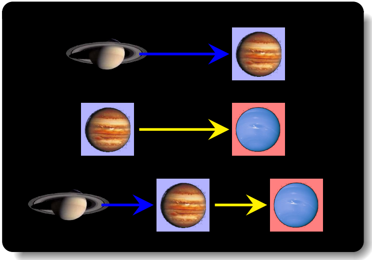

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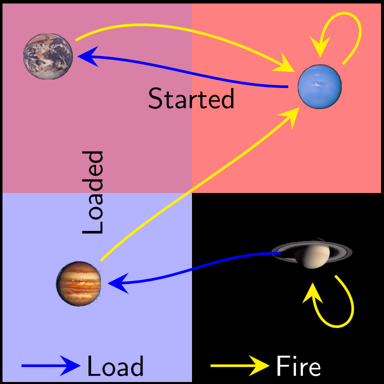

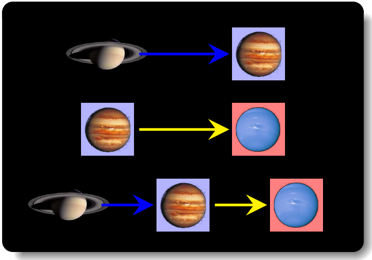

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Introducing the formal language PDL

Propositional dynamic logic

Examples

$$\mathbf{act} = \{\text{load}, \text{fire}\}$$

$$\mathbf{prop} = \{\mathbf{Loaded}, \mathbf{Started}\}$$

- Basic ingredients:
  - A set **act** of actions
  - A set **prop** of propositions
- Action constructions:
  - For building complex actions
  - Composition, iteration, choice, test
- Formula constructions:

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m *will* bring about φ.
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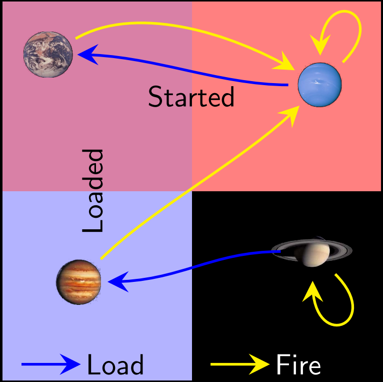
  - $\langle m\rangle\varphi$ — weak dynamic operator

Some PDL examples

Our Universe



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fire *will* make **Started** true.

True:

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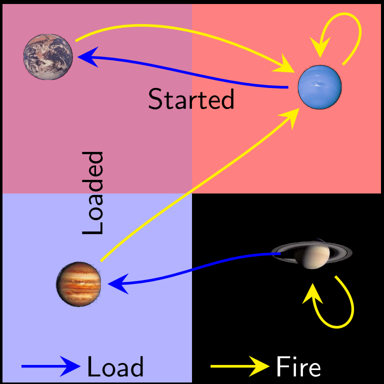

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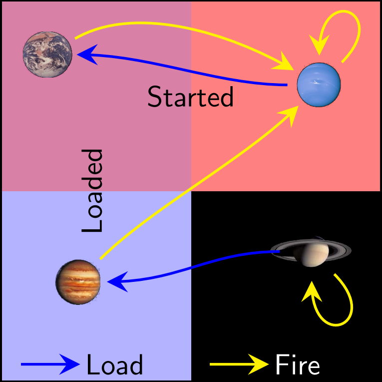

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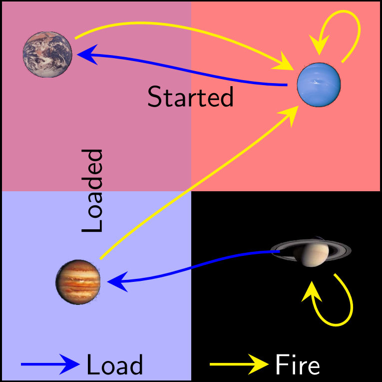

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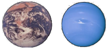
True: 

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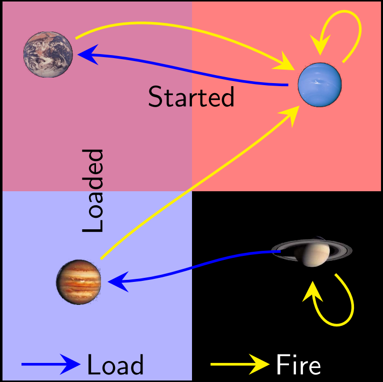

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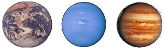
True: 

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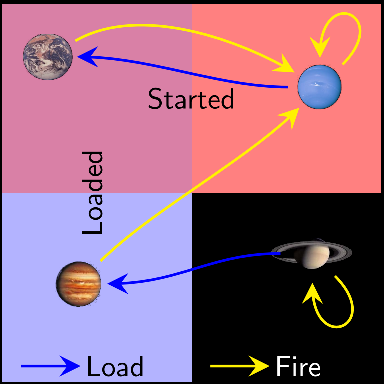

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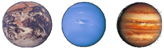
True: 

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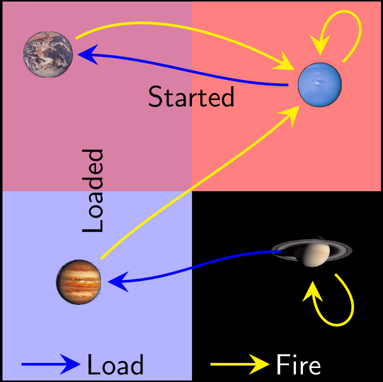

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But *every* world satisfies [load]**Loaded**!

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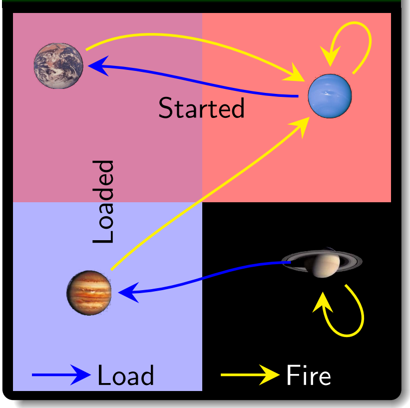

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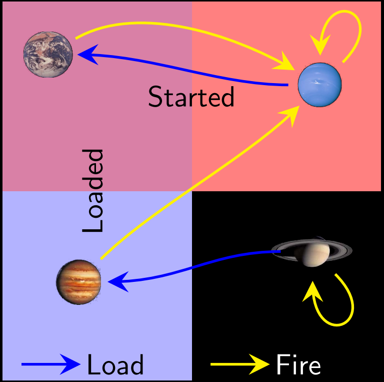

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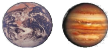
False: 

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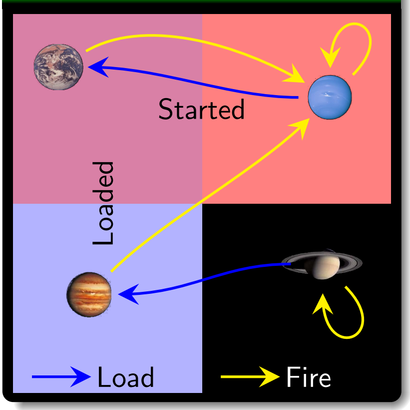

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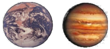

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Actions may have uncertain outcomes.

- Randomness
- Uncertain conditions
- Actions may require skill
- Malfunctioning artifacts

Our models should support *non-determinism*.

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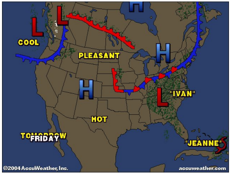

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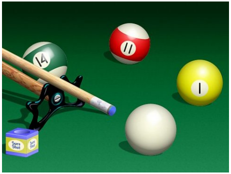

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Adding non-determinism to our model

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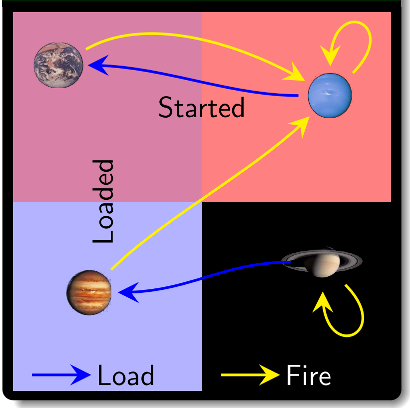

The pistol has a weak spring.

Sometimes, bullet doesn't fire, world doesn't change.

$$[\![\text{fire}]\!]\left(\quad \right) = \left\{ \quad , \quad \right\}.$$

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$$[\![\text{fire}]\!] : \mathcal{W} \to \mathcal{PW}$$

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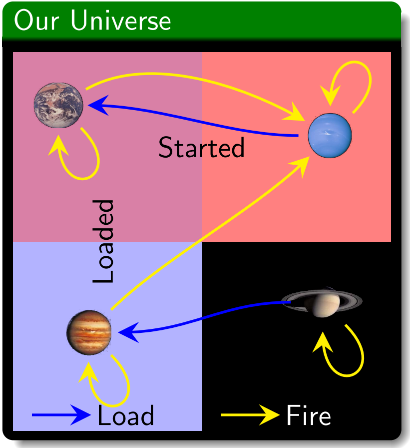

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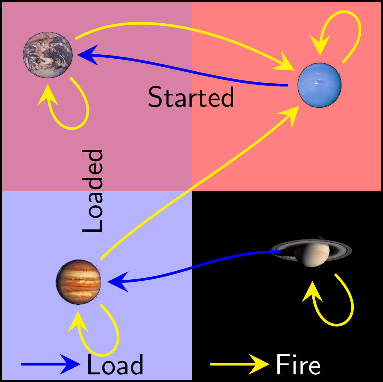

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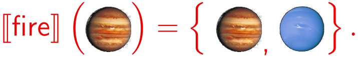

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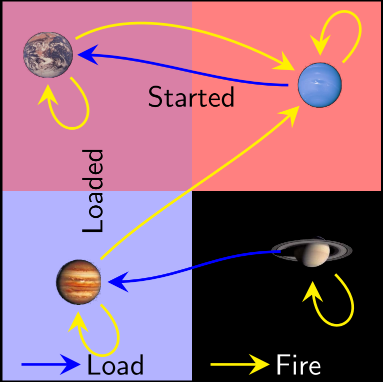

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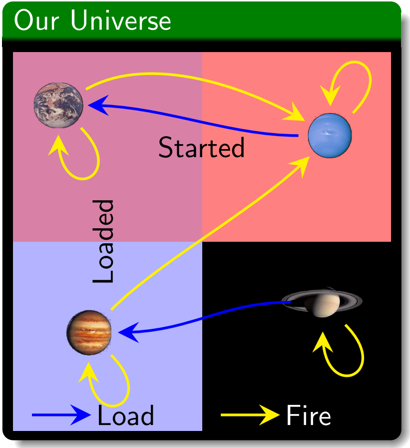

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Ability and modal logic: Kenny's analysis

Ability is closely related to Means-end ascriptions.

Modal logic cannot represent ability (Kenny).

1. $\not\models \varphi \rightarrow \mathsf{Can}\,\varphi$
   - I hit the bull, but I am not able to hit the bull.
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Brief introduction to Brown's logic

But not so fast. . .

Minimal models are weaker than Kripke semantics.

Minimal models

Relevance function: $\alpha : \mathcal{W} \to \mathcal{PPW}$

$w \models \mathrm{Can}\,\varphi$ iff there is $S \in \alpha(w)$ such that S

Intuitively: Each set S in $\alpha(w)$ is an action in w.
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The relation between ability and means

Brown's ability logic is very closely related to our means-end logic.

There is a natural translation of dynamic logic to minimal models.

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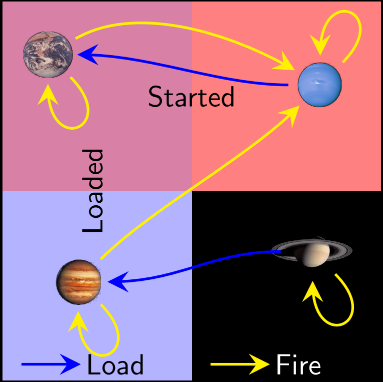
Outline

Efficacy as an essential feature of means

Our Universe



Our picture is unreasonable.

A misfire is less likely than a retort.

We should add probabilities to the picture.

But how?

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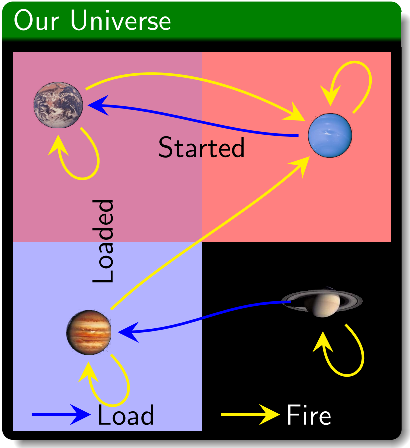

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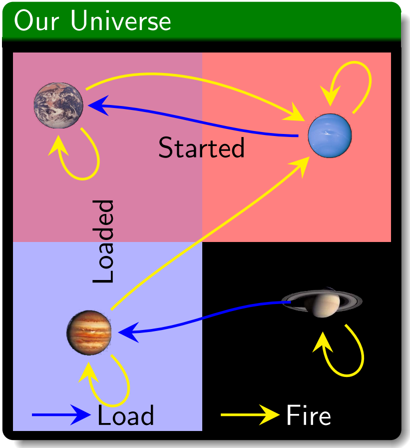

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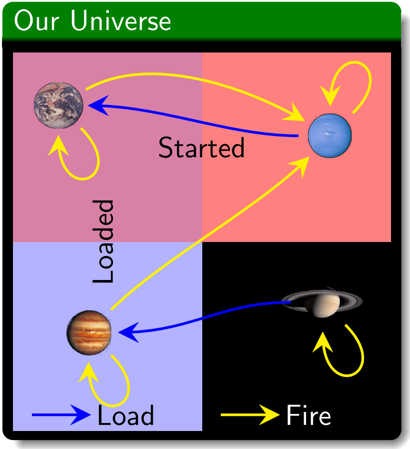

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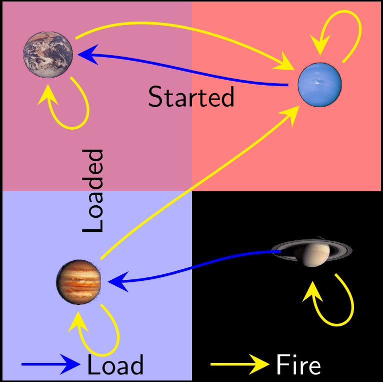

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Different means to same end have different efficacies.

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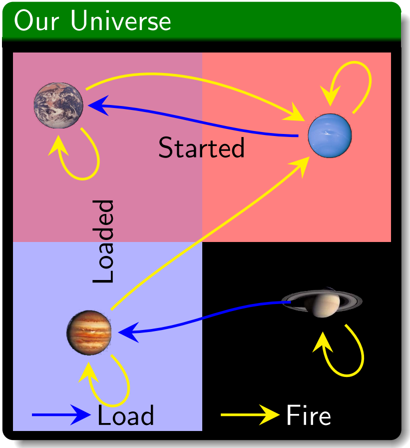

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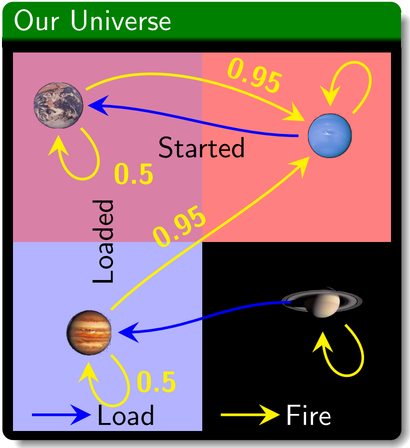

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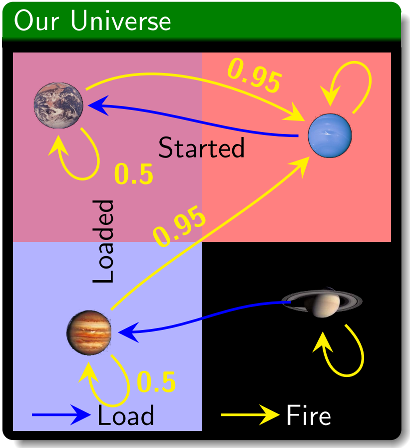

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A brief introduction to fuzzy set theory

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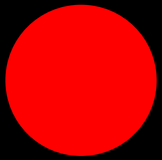

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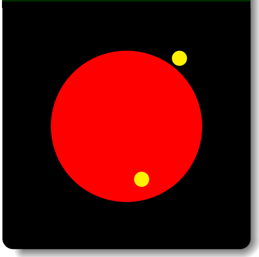

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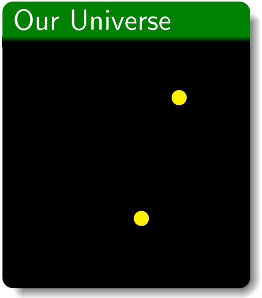


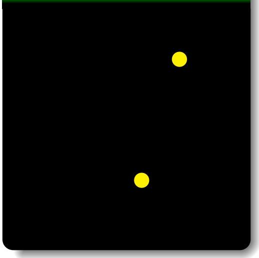
Some propositions aren't so *crisp*.

Fuzzy sets represent ambiguous propositions.

Here, $x \in S$ is assigned some value in $[0, 1]$.

A brief introduction to fuzzy set theory

Our Universe



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Fuzzy sets represent ambiguous propositions.

Here, $x \in S$ is assigned some value in $[0, 1]$.

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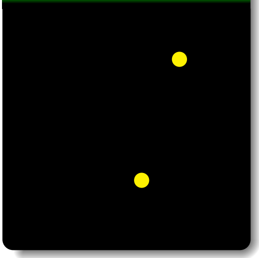

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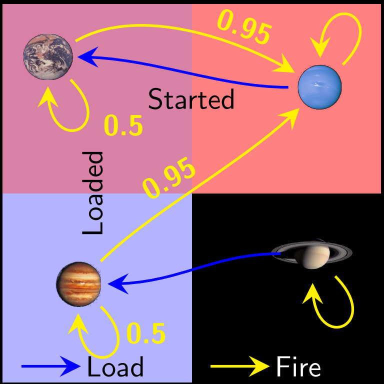

Think again about [fire]**Started**.

That is neither just true nor false.

It's a bit fuzzy.

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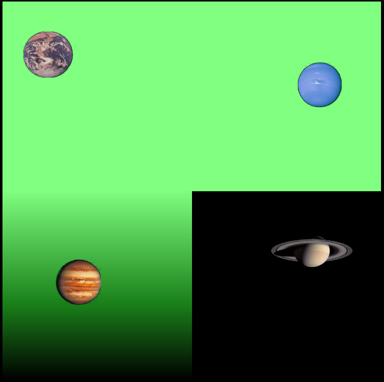

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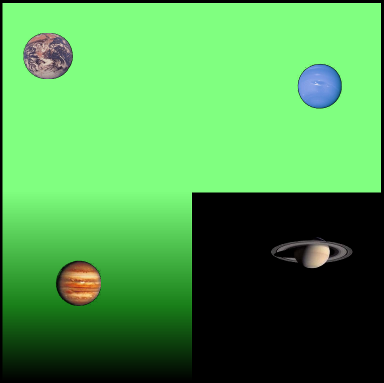

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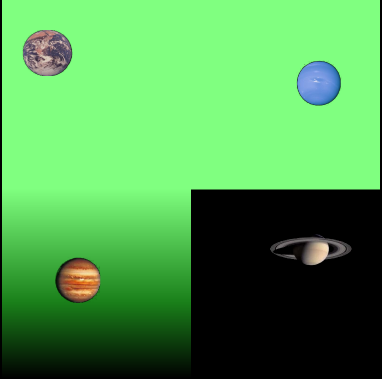

Now, this is a new approach.

There *are* fuzzy modal logics. . . but they're different.

Our fuzzy dynamic logic uses expected values, not conjunctions of implications.

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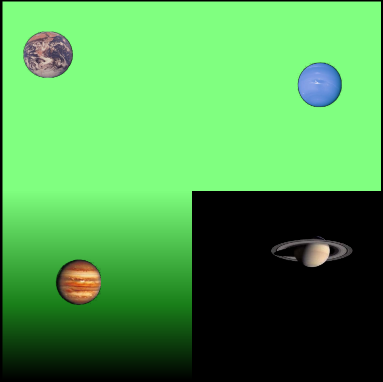

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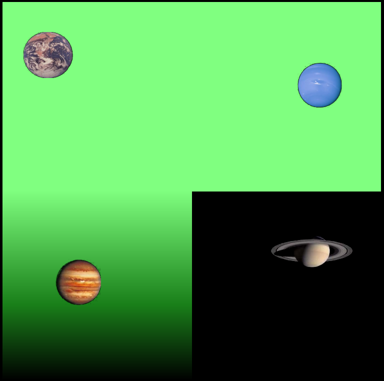

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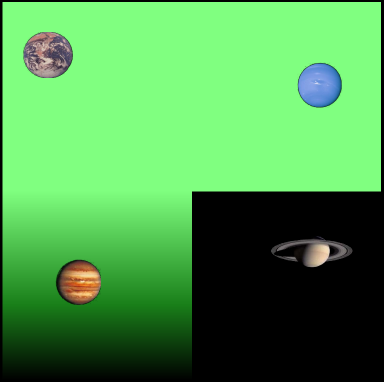

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The Main Issues

- The relationship between ability and means.
- Fuzzy sets and dynamic logic.
- Conditions and means-chaining.
- Back to artifactual functions.

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