A Semantics for Means-End Relations

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May 19, 2005

Outline

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1. Means-end relations in practical reasoning
   - von Wright's example
   - Initial informal analysis
2. PDL and sufficient means
   - Introduction to PDL
   - Sufficient means-end relations

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   - von Wright's necessary means
   - Involvement

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The conclusion is an *action* or an *intention*.

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A working example from von Wright.

I want to make the hut habitable.

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- Concludes in a *necessary* action.

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We focus on the semantics of means-end relations.

Initial steps for a means-end semantics

- An end is some desirable condition – a *proposition*.

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Intuition for $[\alpha]\varphi$: After doing α, φ will hold.

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$w \xrightarrow{\alpha} w'$ means: one can reach w' by doing α in w.

$w \models [\alpha]\varphi$ *iff* whenever $w \xrightarrow{\alpha} w'$, then $w' \models \varphi$.

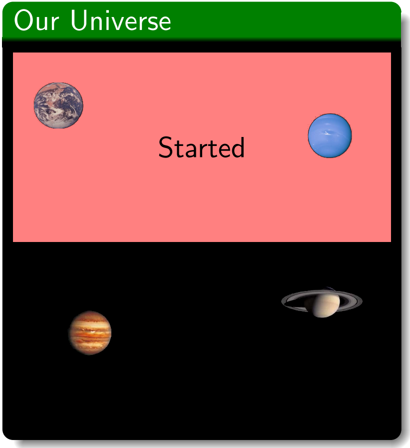
$w \models \langle\alpha\rangle\varphi$ *iff* there is $w \xrightarrow{\alpha} w'$ such that $w' \models \varphi$.

A simple example of possible worlds

Our Universe

A set of worlds involving a footrace and starter pistol.

A simple example of possible worlds



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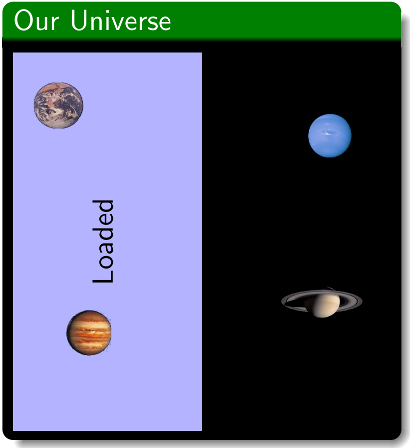

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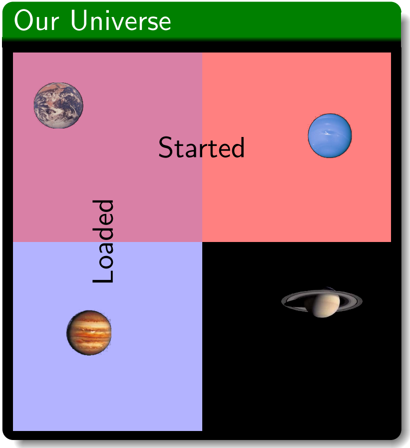

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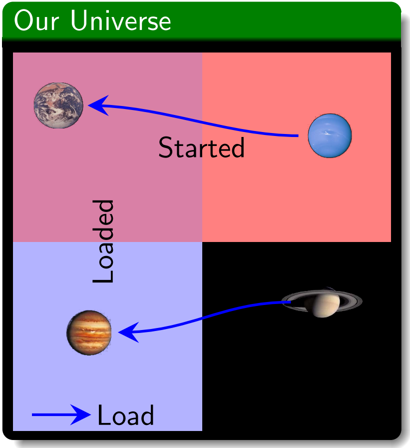

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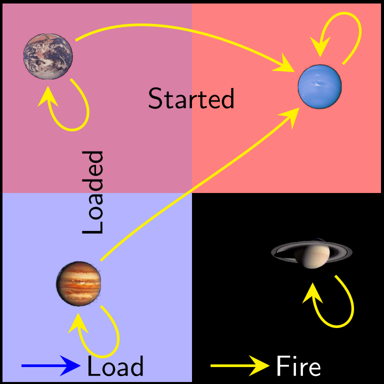

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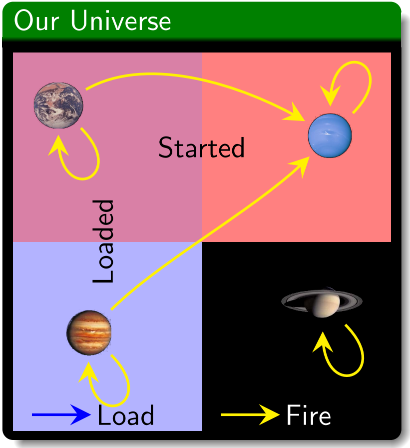

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Note: "Fire" means "pull trigger". We allow misfires.

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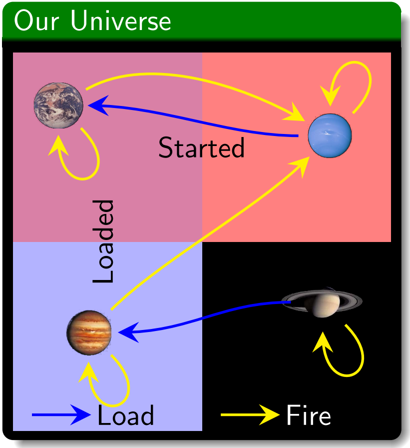

$\langle \text{fire} \rangle$**Started**

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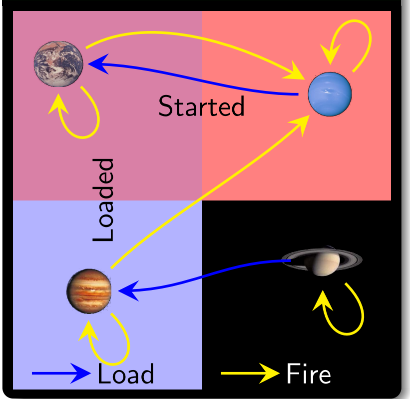

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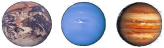

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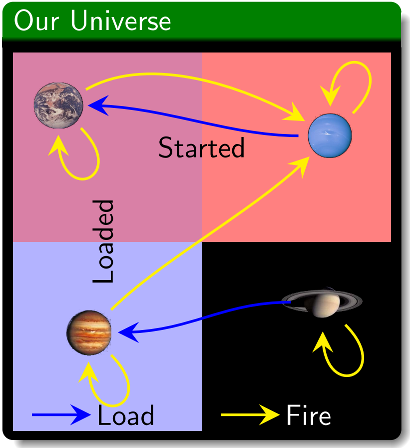

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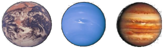
True: 

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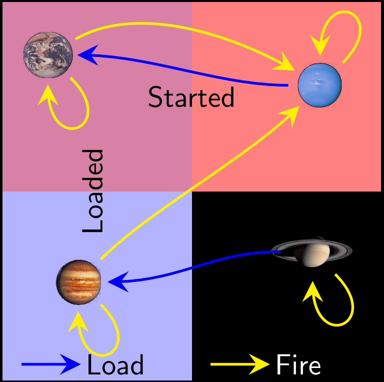

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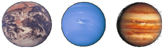
True: 

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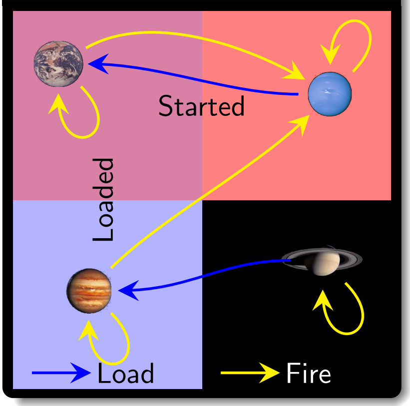

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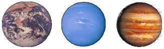
True: 

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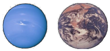
True: 

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Sufficient means-end relations

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Doing α in *w* will yield φ

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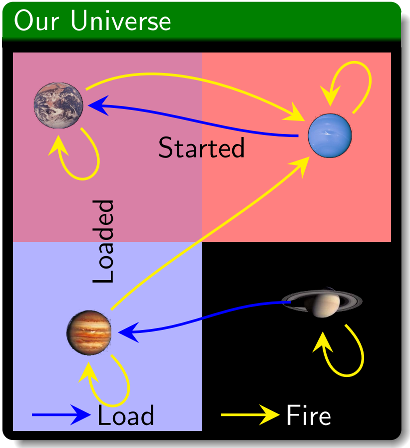
But what if $\alpha = \alpha_1; \alpha_2$?

A necessary means-end relation



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A necessary means-end relation

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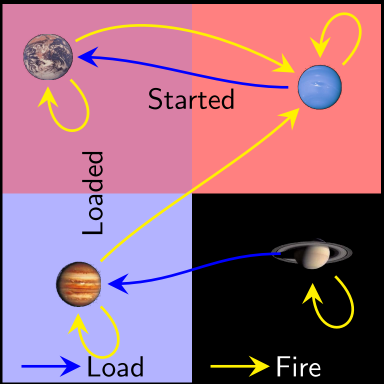

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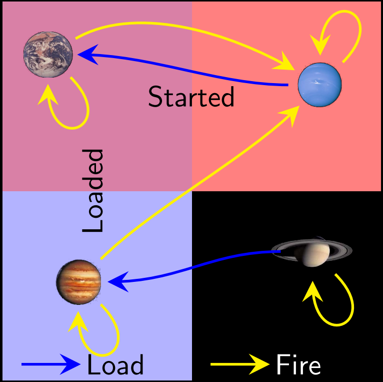

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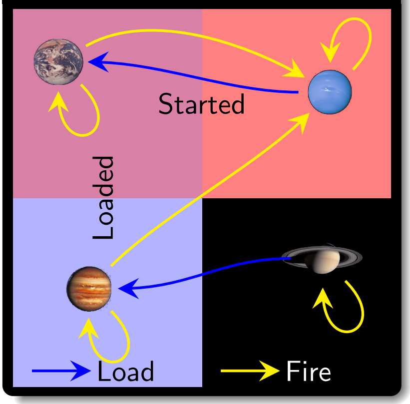

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Does fire; load; fire; fire involve load; fire? Yes!

Does skip; load; skip; fire involve load; fire?

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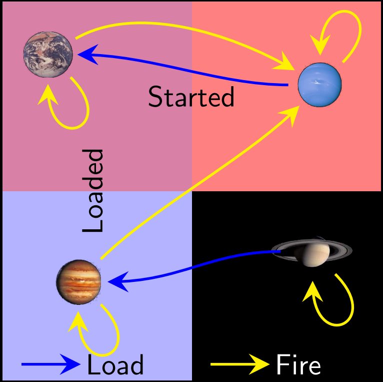

In , load; fire is a necessary means to **Started**.

Does fire; <u>load; fire</u>; fire involve load; fire? Yes!

Does skip; <u>load</u>; skip; <u>fire</u> involve load; fire? Also yes!

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- Non-deterministic choice $\cup$ is the join for $\preccurlyeq$.
- If $\beta \preccurlyeq \alpha$, then $\beta;\gamma \preccurlyeq \alpha;\gamma$ and $\gamma;\beta \preccurlyeq \gamma;\alpha$.

Doing α and involvement

Necessary means:

If α is a *necessary means* to φ, then

- φ will not be realized without *doing* α and
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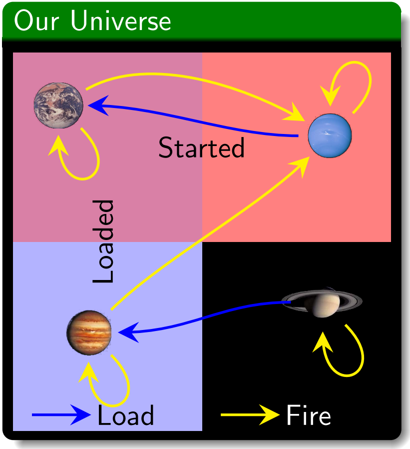
Examples of necessary means to **Started**



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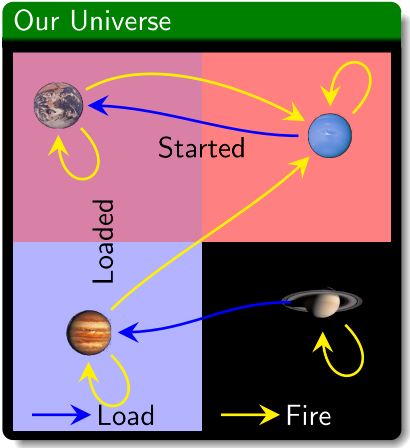

	necessary in ...
fire	 
load	

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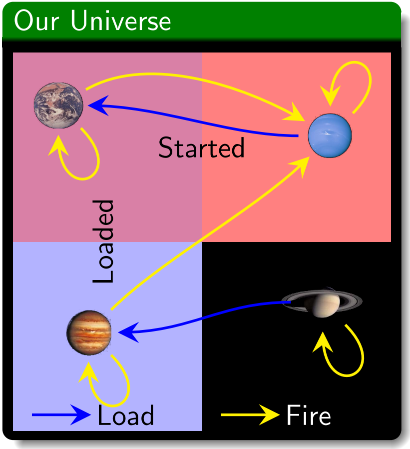

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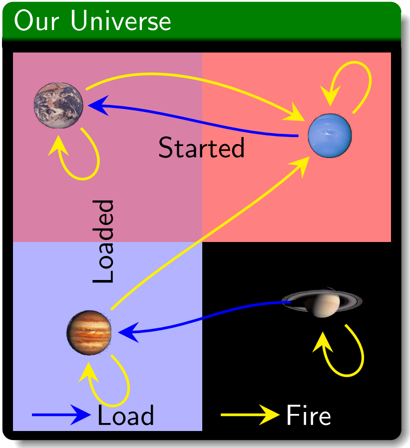

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To realize **Started**,
one must do some β involving
every necessary means.

Additional topics

- Involvement with test actions (in paper).

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- From means-end relations to artifactual functions.

Outline

4. Conditionals and the frame problem
   - Local vs. conditional relations
   - Non-monotonicity

Local means-end relations

Our definition

In w, α is a sufficient means to φ iff $w \models [\alpha]\varphi$ & $\langle\alpha\rangle$**True**.

This is a very narrow sense of means-end relation.

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- every "normal" world in which we are in Eindhoven?

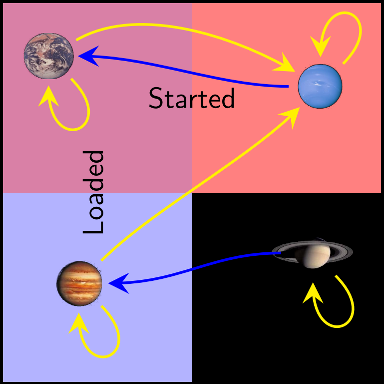
Introducing conditional means-end relations

Conditional relation:

Assuming ψ,

α is a means to φ.

Our Universe



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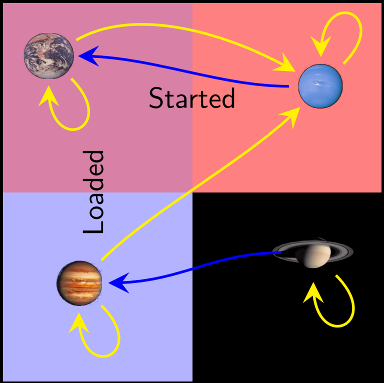

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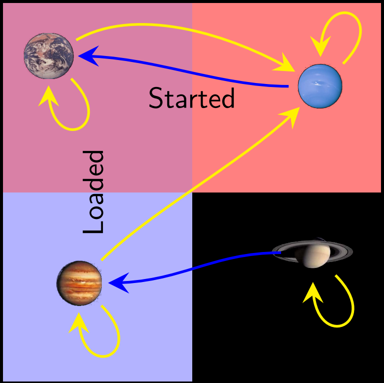

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α is a local means to φ.

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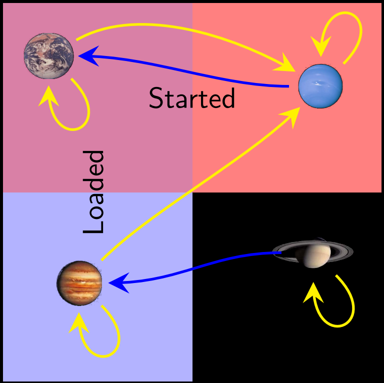

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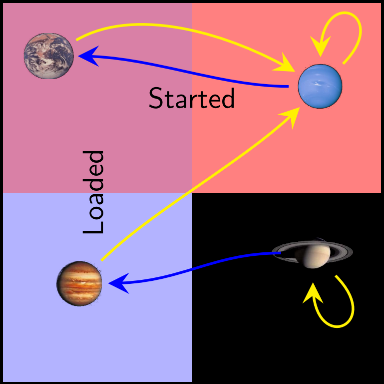

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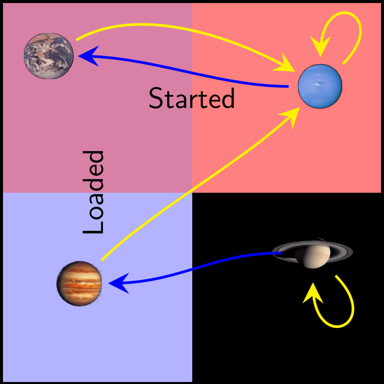

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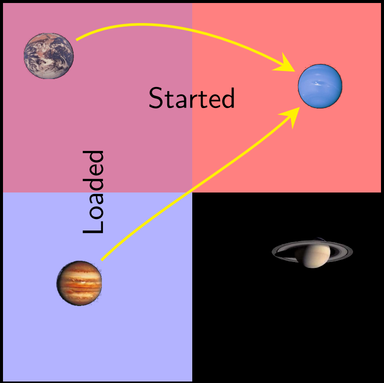

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(or "Why means-end reasoning is hard")

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If I had money, she would marry me.

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Problem: If I rob her, she will hate me and
(**money** & **HATE**) $\not\to$ [propose]**marry**.

Our conditional should be non-monotonic

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