A Semantics for Means-End Relations

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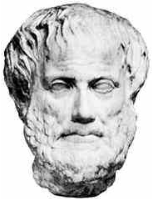
Practical Reasoning

Practical reasoning is concerned with actions to attain desired results.



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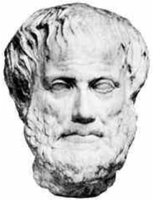

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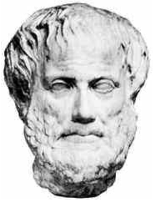

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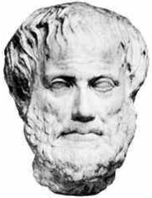

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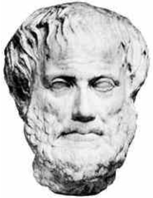

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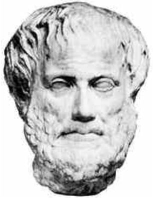
The conclusion is an *action* or an *intention*.



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Note: distinct premises

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- How to evaluate the syllogism?
- How do the premises make the conclusion necessary?
- For this, we need to know the meaning of the premises.

Our project

Aim: Formal semantics for means-end relations

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Outline

1. Means-end relations in PDL
   - A brief overview of PDL
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Think possible worlds!

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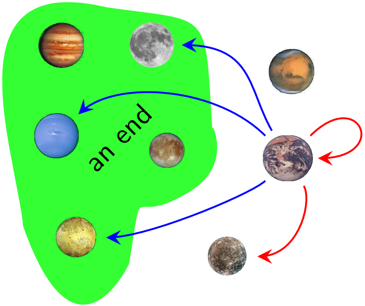

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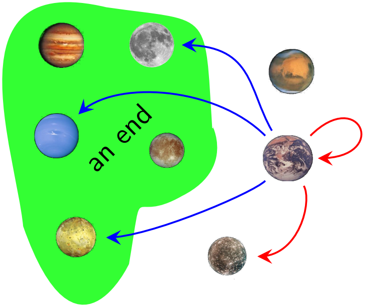

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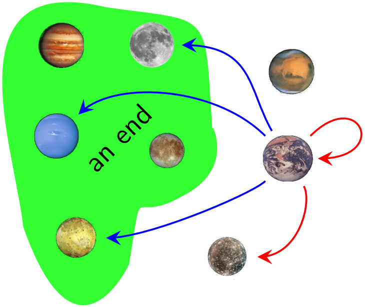

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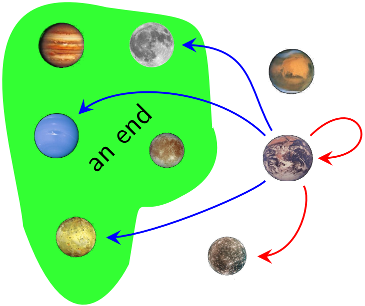

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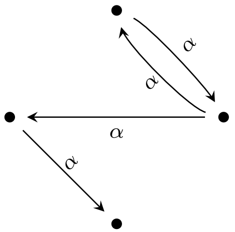
- $\langle\alpha\rangle\varphi$: after doing α, φ *might* hold.

PDL semantics



Possible world semantics with transition systems for each action α.

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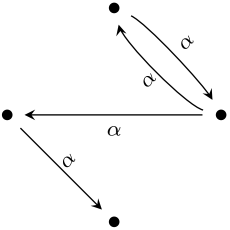

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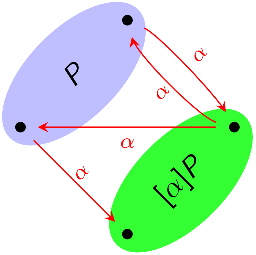

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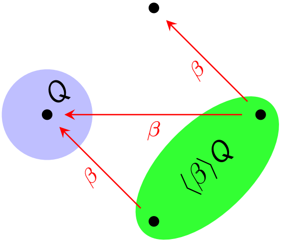

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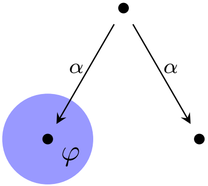

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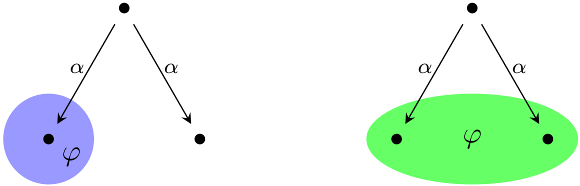

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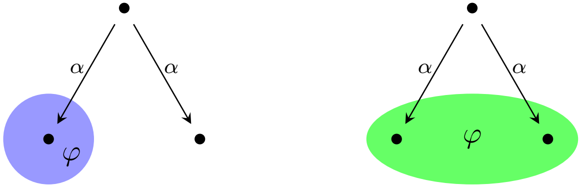

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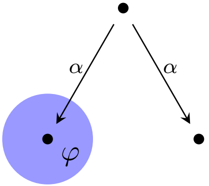

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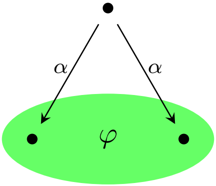

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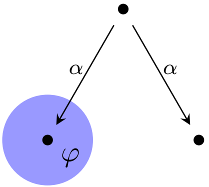

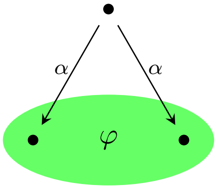

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Caveat: This definition omits relevance.

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- Give up distributivity.
- Gain richer sense of "using" objects.

PDL means-end relations are *local* relations

Our definition

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Solution:

- add a *non-monotonic* conditional operator to PDL.

Means distinguished by efficacy

Different means to a common end have different degrees of reliability.

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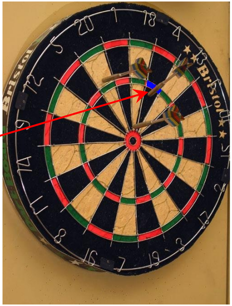

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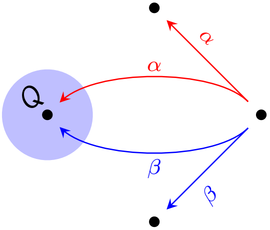
Efficacy: The degree of reliability of a means to an end.

From non-determinism to probabilities



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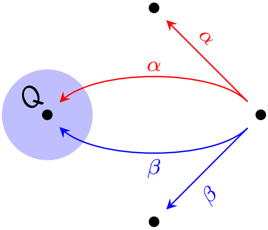

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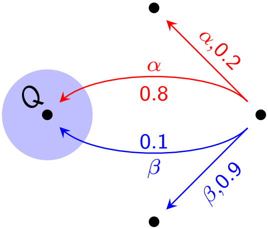

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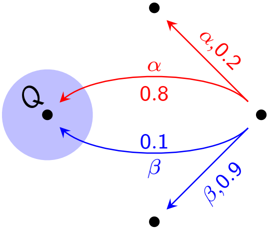

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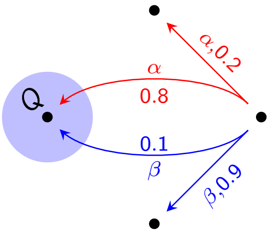

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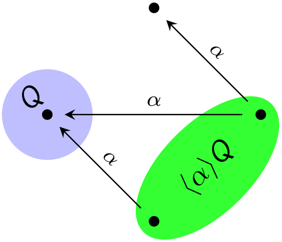
Interpret $\langle\alpha\rangle$ as a *fuzzy* operator.

Reliability as a fuzzy proposition



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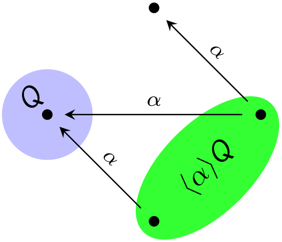

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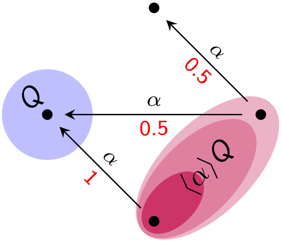

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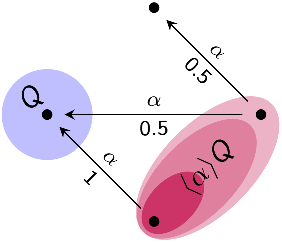

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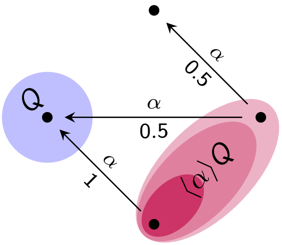

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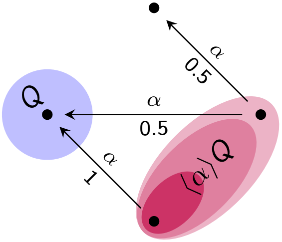

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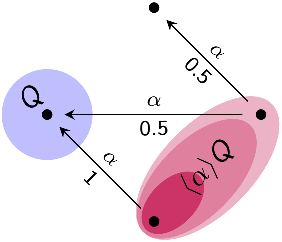

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- Elegant treatment of complex ends, like $\langle\alpha\rangle\varphi \wedge \langle\beta\rangle\psi$.

Concluding remarks

Summary:

- Semantics for means-end relations

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Thank you.

Outline

Reevaluating material implication
(or "Why means-end reasoning is hard")

A simple derivation:

If I had money, she would marry me.

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Problem: If I rob her, she will hate me and
(**money** & **HATE**) $\not\rightarrow$ [propose]**marry**.

Our conditional should be non-monotonic

Non-monotonicity

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Reasoning about means is hard.

Outline

3 Non-monotonicity

4 Extra details on fuzzy PDL
- Probability is not fuzziness
- Fuzzy ends

But probability $\neq$ fuzziness...

Slogan: Probabilities and fuzziness are different.

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Truth degrees

"Probably φ": $P(\varphi)$

$\langle\alpha\rangle\varphi$: $\sum_{w' \in \mathcal{W}} P(w \xrightarrow{\alpha} w') \cdot [\![\varphi]\!](w')$

Fuzzy ends

An accidental advantage

Weapons are for causing harm.

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Examples: slingshot, nuke

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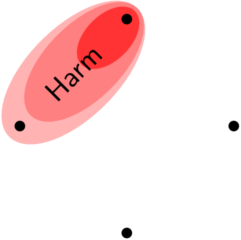

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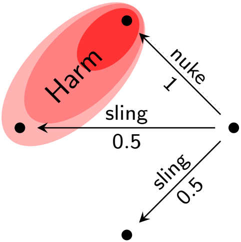

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This end is fuzzy.

Fuzzy PDL allows for fuzzy ends.

A nuke is more effective in causing harm than a slingshot.

(Duh.)