The Muddy Children:
A logic for public announcement

Jesse Hughes

Eindhoven University of Technology

February 16, 2007

Outline

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The muddy children

Quincy

The muddy children

Quincy

Prescott

The muddy children

Quincy

Prescott

Hughes

The muddy children

Quincy

Prescott

Hughes

Baba: “At least one of you is muddy.”

The muddy children

Quincy

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Baba: “At least one of you is muddy.”

Baba: “Are you muddy?”

The muddy children

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Quincy: “I don’t know.”
Prescott: “I don’t know.”
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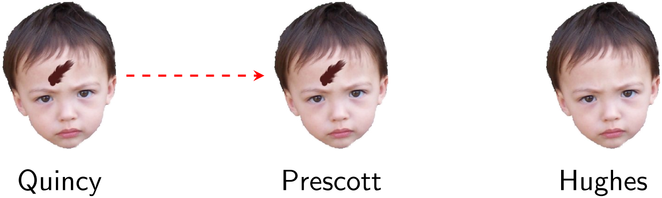
The muddy children



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The muddy children

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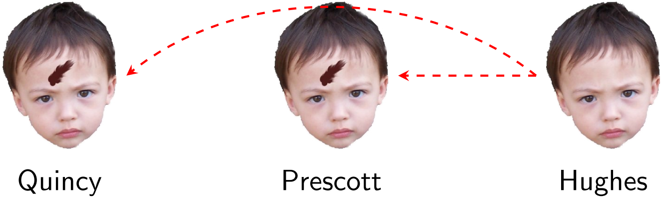

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The muddy children

Quincy Prescott Hughes

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Quincy: "Prescott would have said 'yes' last time!"

The muddy children


Quincy


Prescott


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Baba: "Are you muddy?"
Quincy: "Aha! What if I <u>wasn't</u> muddy?"
Quincy: "Then Prescott would not have seen any muddy kids."
Quincy: "Prescott would have said 'yes' last time!"
Quincy: "I must be muddy."

The muddy children

Quincy

Prescott

Hughes

Baba: “Are you muddy?”
Quincy: “Yes.”
Prescott: “Yes.”
Hughes: “I don’t know.”

The muddy children

Quincy

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Hughes

When Baba said, “At least one kid is muddy,” every kid knew that. . .

The muddy children

Quincy

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When Baba said, “At least one kid is muddy,” every kid knew that. . . but they didn’t know that the other kids knew that!

The muddy children

Quincy

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When Baba said, “At least one kid is muddy,” every kid knew that. . . but they didn’t know that the other kids knew that!

Public announcements of φ tell you φ, everyone knows φ, everyone knows that everyone knows φ, . . .

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Modal operators

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It ought to be φ.	$O\varphi$	$P\varphi$	φ is permitted.
I know φ.	$K\varphi$	??	I think φ is possible.

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Kripke semantics

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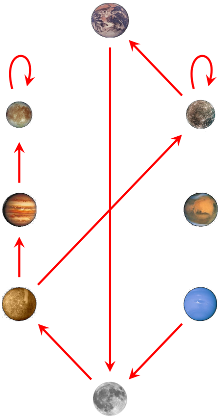
Let $\mathcal{W}$ be a set of worlds with a graph.



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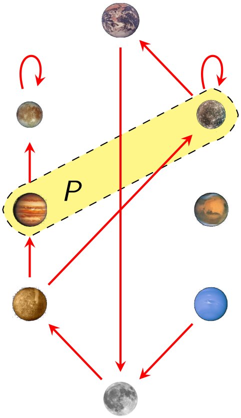
Write $w \models P$ if P is true at world w.



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\begin{aligned}
w \models \varphi \wedge \psi \quad &\text{iff} \quad w \models \varphi \text{ and } w \models \psi \\
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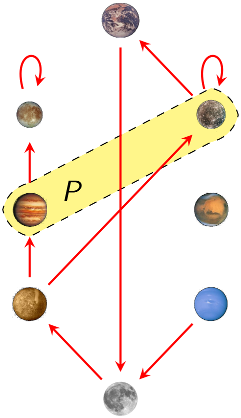

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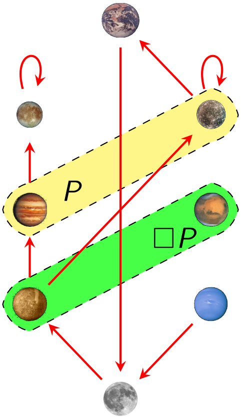

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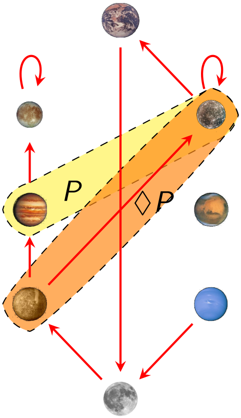
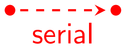


Modal axioms and frame conditions

Axioms on $\Box$ correspond to conditions on the graph.

Name	Axiom	Graph is...
(D)	$\Box\varphi \rightarrow \Diamond\varphi$	serial



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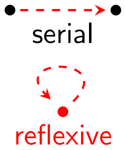

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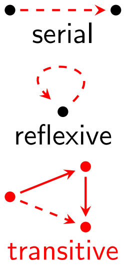

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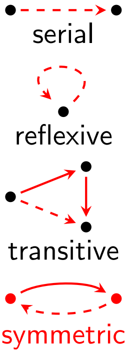

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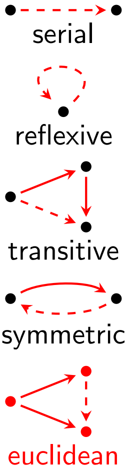

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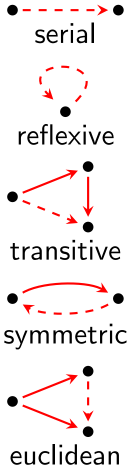
If $\Box$ satisfies (M), (4) and (B), then the graph is an equivalence relation.



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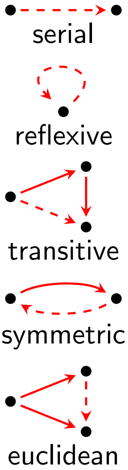

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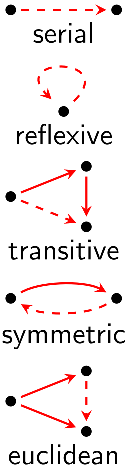
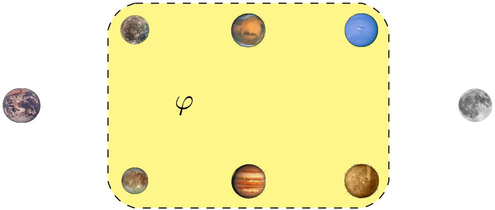
- Write w —— w'.
- Don't bother to draw loops.



Outline

The epistemic operator



For each agent α, we introduce an operator K_α.

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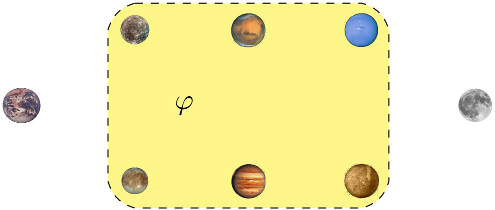

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$K_\alpha\varphi$ means "α knows φ."

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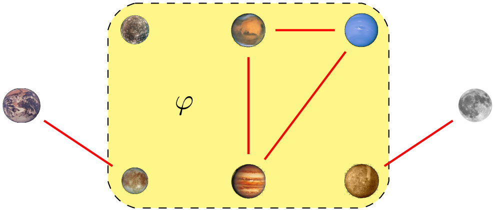

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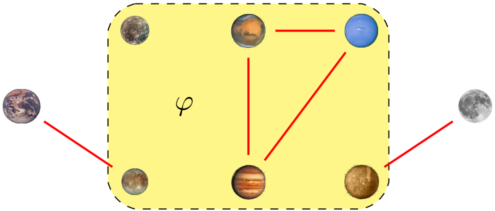

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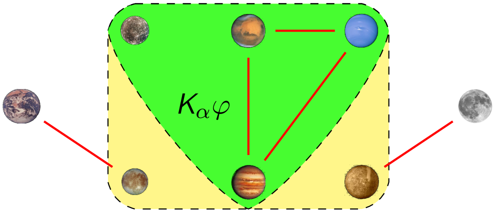

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Each α has its own graph, too.
An edge $w \overset{\alpha}{\text{———}} w'$ means "α can not distinguish w from w'."
$w \models K_\alpha\varphi$ iff for every $w \overset{\alpha}{\text{———}} w'$, $w' \models \varphi$.

More on K_α

Quincy

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More on K_α

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$K_\alpha\varphi$ means "α knows φ."

What does $\neg K_\alpha \neg\varphi$ mean?

More on K_α

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$K_\alpha\varphi$ means "α knows φ."
What does $\neg K_\alpha \neg\varphi$ mean? α considers that φ is possible.

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$K_\alpha \varphi$ means "α knows φ."

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What about $K_\alpha K_\beta \varphi$?

More on K_α

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$K_\alpha \varphi$ <u>means</u> “α knows φ.”

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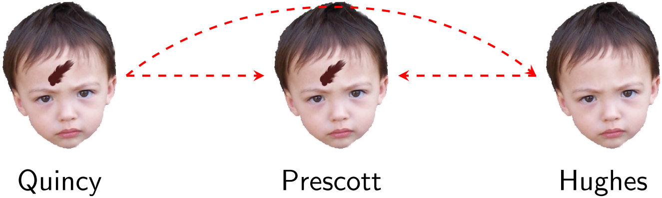
More on K_α



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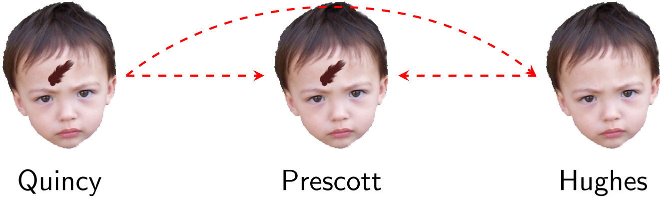
For instance, Quincy knows that Hughes knows that Prescott is muddy.

More on K_α



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What about $K_\alpha K_\beta \varphi$? α knows that β knows that φ.
For instance, Quincy knows that Hughes knows that Prescott is muddy. In other words, $K_Q K_H(P \text{ is muddy})$.

Properties of K_α

Quincy Prescott Hughes

$K_\alpha \varphi \rightarrow \varphi$ knowledge is true

Properties of K_α

$K_\alpha \varphi \to \varphi$ knowledge is true

$K_\alpha \varphi \to K_\alpha K_\alpha \varphi$ positive introspection

Properties of K_α

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$K_\alpha \varphi \rightarrow \varphi$	knowledge is true
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$\neg K_\alpha \varphi \rightarrow K_\alpha \neg K_\alpha \varphi$	negative introspection

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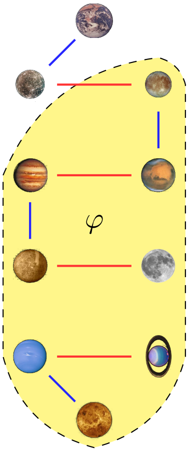
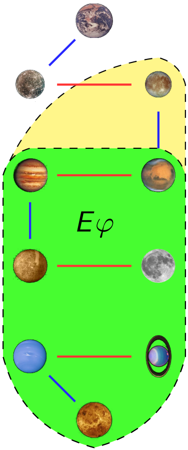
$K_\alpha \varphi \to \varphi$	knowledge is true
$K_\alpha \varphi \to K_\alpha K_\alpha \varphi$	positive introspection
$\neg K_\alpha \varphi \to K_\alpha \neg K_\alpha \varphi$	negative introspection
$K_\alpha(\varphi \to \psi) \to (K_\alpha \varphi \to K_\alpha \psi)$	distributivity

Universal and common knowledge



Universal and common knowledge

- Universal knowledge ($E\varphi$):
  - Everyone knows φ.
  - No one-step paths outside of φ.



Universal and common knowledge

- Universal knowledge ($E\varphi$):
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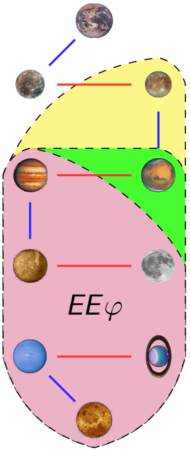
- Universal knowledge of universal knowledge ($EE\varphi$):
  - Everyone knows that everyone knows φ.
  - No two-step paths outside of φ.
  - No one-step paths outside of universal knowledge.



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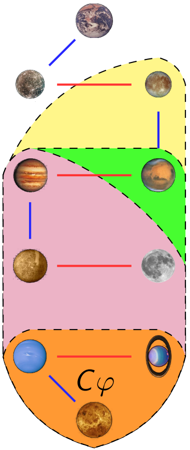
  - Everyone knows that everyone knows φ.
  - No two-step paths outside of φ.
  - No one-step paths outside of universal knowledge.
- Common knowledge ($C\varphi$):
  - Everyone knows that everyone knows that. . . that everyone knows φ.
  - No paths out of φ.



Back to the kids

Eight possible worlds.

Back to the kids

Eight possible worlds.

0 - clean

1 - muddy

Back to the kids

Q	P	H
0	0	0

Eight possible worlds.

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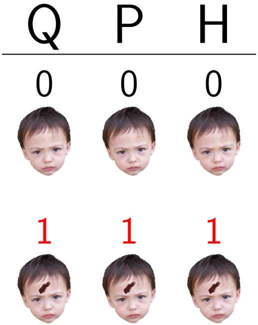
Back to the kids



Eight possible worlds.

0 - clean

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Back to the kids

Q	P	H
0	0	0

1	1	1

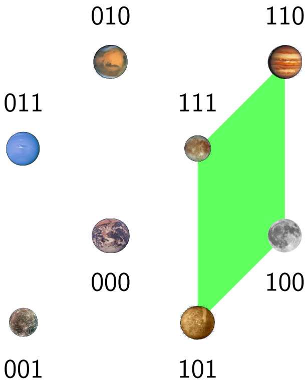
1	1	0

Eight possible worlds.

0 - clean

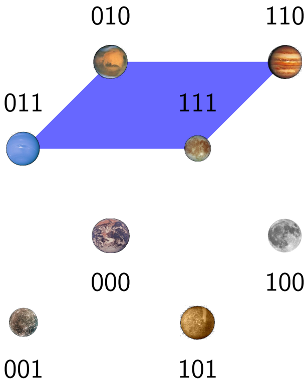
1 - muddy

Back to the kids



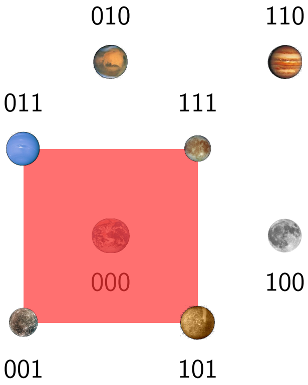
Quincy is muddy in these four worlds.

Back to the kids



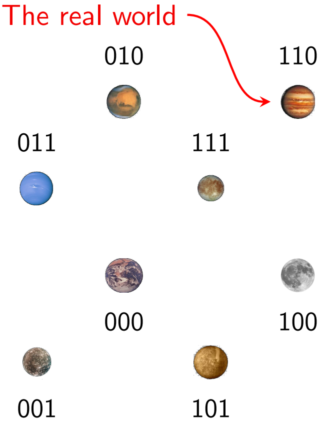
Quincy is muddy in these four worlds.
Prescott is muddy in these four.

Back to the kids



Quincy is muddy in these four worlds.
Prescott is muddy in these four.
And Hughes is muddy in these four.

Back to the kids



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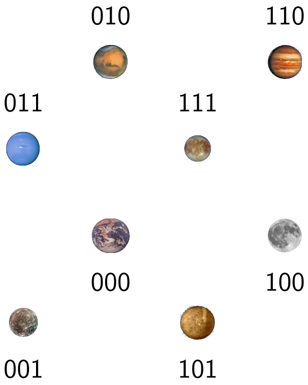
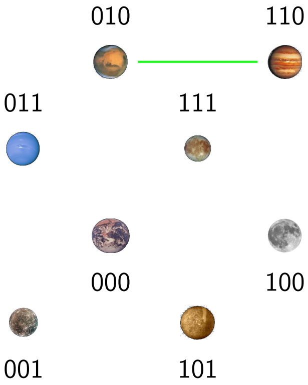


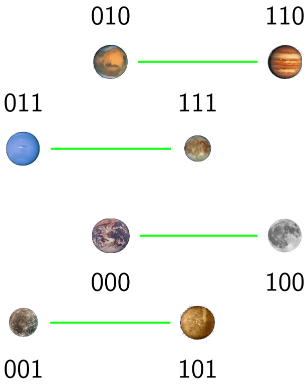
Quincy cannot distinguish a world where he is muddy from one where he isn't.

Back to the kids



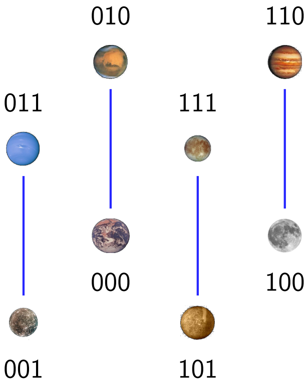
Quincy cannot distinguish a world where he is muddy from one where he isn't.
World 110 is indistinguishable from 010.

Back to the kids



Quincy cannot distinguish a world where he is muddy from one where he isn't.
World 110 is indistinguishable from 010.
Quincy's epistemic relation.

Back to the kids



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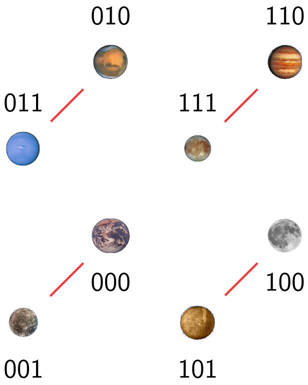
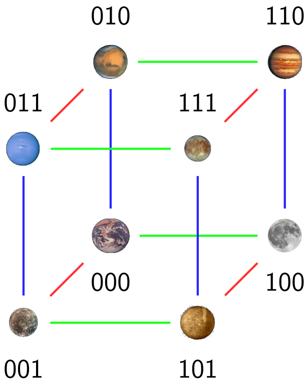
Quincy's epistemic relation.
Prescott's relation.

Back to the kids



Quincy cannot distinguish a world where he is muddy from one where he isn't.
World 110 is indistinguishable from 010.
Quincy's epistemic relation.
Prescott's relation.
And Hughes's relation.

Back to the kids



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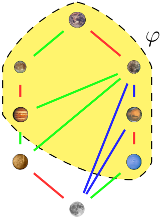
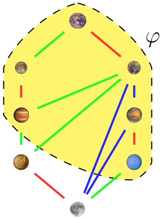
Outline

Dynamic features



What happens when someone announces φ?

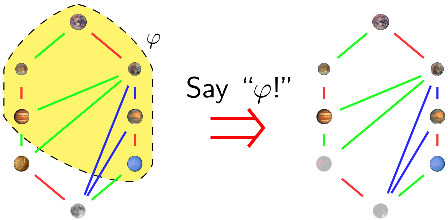
Dynamic features



What happens when someone announces φ?

Everyone learns that φ was true when announced.

Dynamic features



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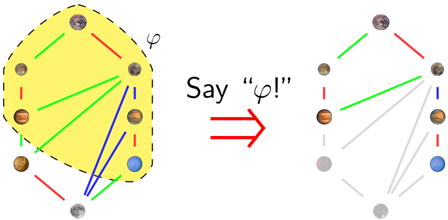

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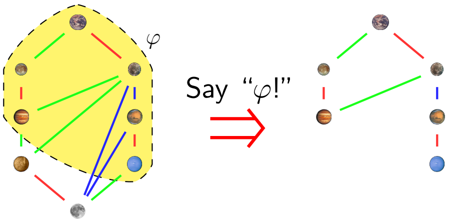

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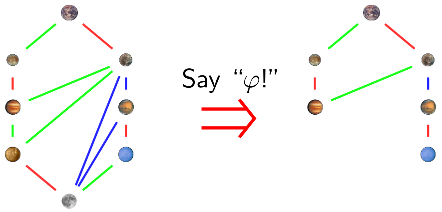
Edges, too!

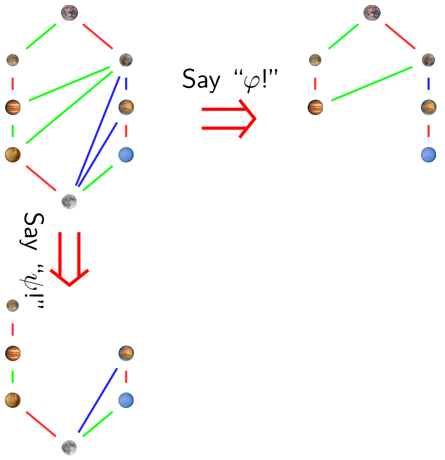
Information reduces uncertainty by eliminating possibilities.

A model of possible models!



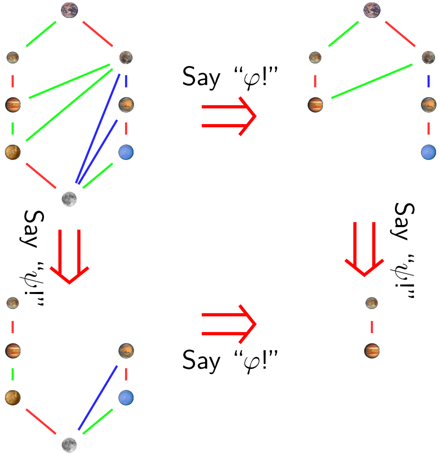
- Announcing φ changes the model.

A model of possible models!



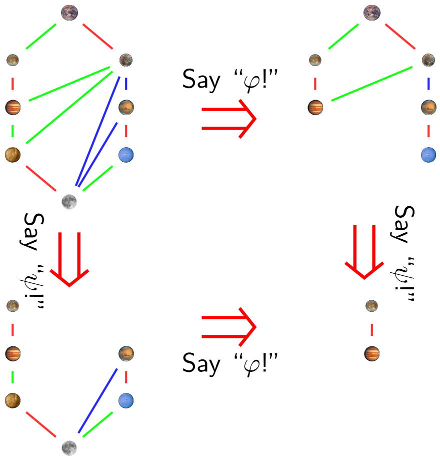
- Announcing φ changes the model.
- Announcing ψ changes it another way.

A model of possible models!



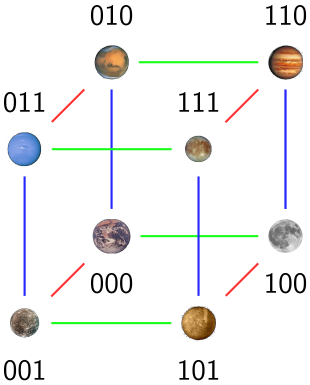
- Announcing φ changes the model.
- Announcing ψ changes it another way.
- Get a transition system on models.

A model of possible models!



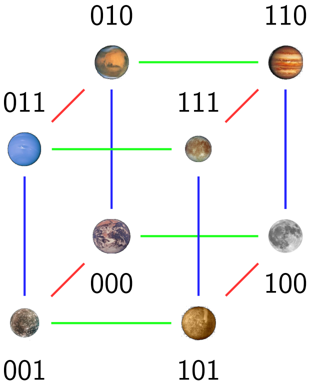
- Announcing φ changes the model.
- Announcing ψ changes it another way.
- Get a transition system on models.
- Another Kripke frame!

Resolving the muddy children



Baba: "At least one of you is muddy."

Resolving the muddy children



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World 000 is inconsistent with this announcement.

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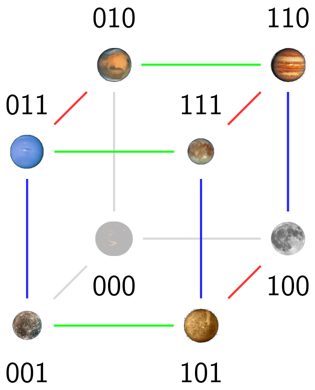
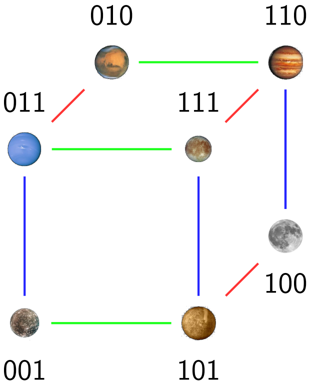


Baba: “At least one of you is muddy.”
World 000 is inconsistent with this announcement.
We remove it from the model.

Resolving the muddy children



Baba: "At least one of you is muddy."
World 000 is inconsistent with this announcement.
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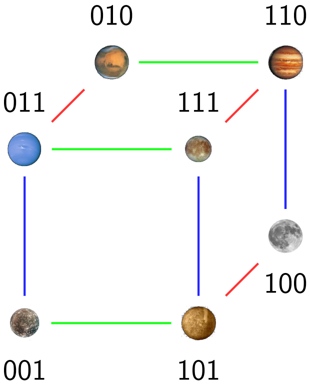
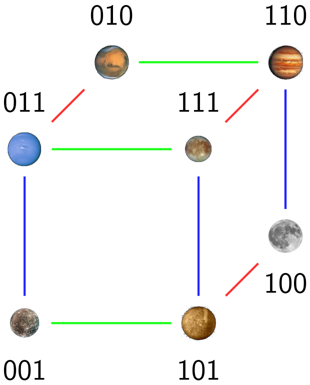
Before $w_{110} \models E\varphi$.

Resolving the muddy children



Baba: "At least one of you is muddy."
World 000 is inconsistent with this announcement.
We remove it from the model.
Before $w_{110} \models E\varphi$.
Now $w_{110} \models C\varphi$.

Resolving the muddy children



Baba: "Are you muddy?"
Quincy: "I don't know."
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Hughes: "I don't know."

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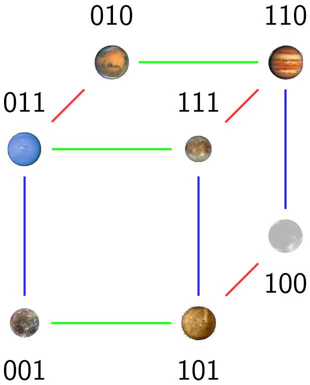

Baba: “Are you muddy?”
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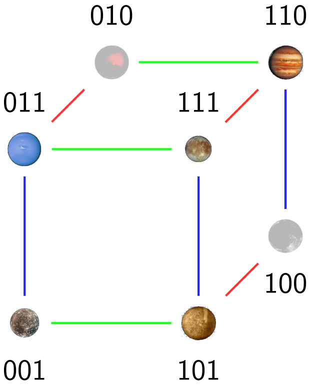
Remove world 100!

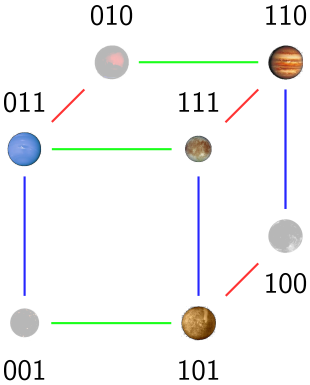
Resolving the muddy children



Baba: “Are you muddy?”
Quincy: “I don’t know.”
Prescott: “I don’t know.”
Hughes: “I don’t know.”

Remove world 100!
Remove world 010!

Resolving the muddy children



Baba: "Are you muddy?"
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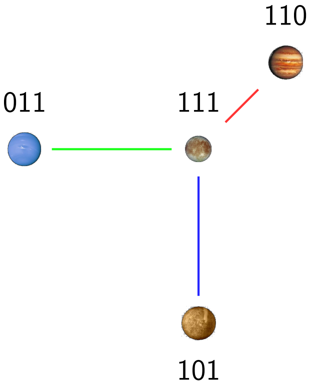
Remove world 100!
Remove world 010!
Remove world 001!

Resolving the muddy children



Baba: "Are you muddy?"
Quincy: "I don't know."
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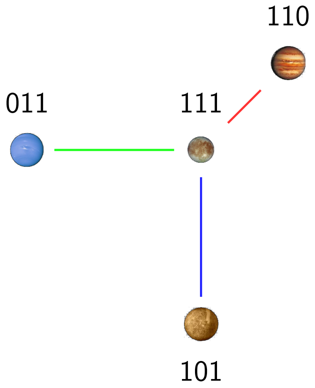
Remove world 100!
Remove world 010!
Remove world 001!

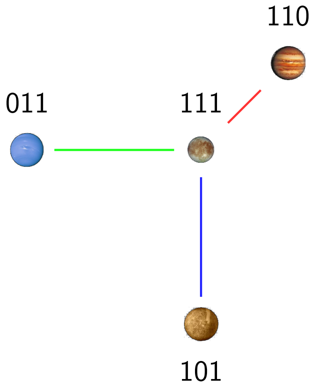
A much simpler model!

Resolving the muddy children



But now:
$w_{110} \models K_Q(\text{"}Q\text{ is muddy"})$!

Resolving the muddy children



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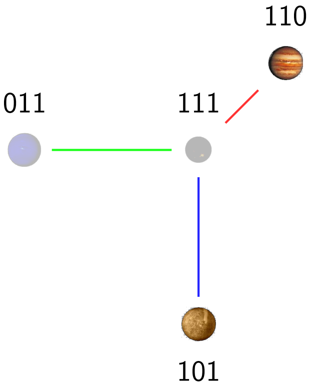

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$w_{110} \models K_Q(\text{"}Q \text{ is muddy"})$!

Baba: "Are you muddy?"

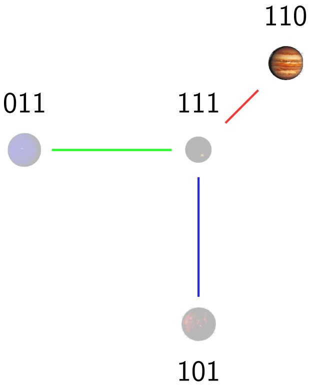
Quincy: "Yes!"

Prescott: "Yes!"

Hughes: "I don't know."

Quincy knows Quincy is muddy: remove 011 and 111.

Resolving the muddy children



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Baba: “Are you muddy?”

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Prescott: “Yes!”

Hughes: “I don’t know.”

Quincy knows Quincy is muddy: remove 011 and 111.

Prescott knows Prescott is muddy: remove 111 and 101.

Resolving the muddy children

110

Baba: "Are you muddy?"
Quincy: "Yes!"
Prescott: "Yes!"
Hughes: "No!"

References

- Stanford Encyclopedia of Philosophy. . http://plato.stanford.edu/entries/logic-modal/
- Benthem, J. v. "Language, logic, and communication". In Logic in Action, J. van Benthem, et al. ILLC, 2001.
- Benthem, J. v. "One is a Lonely Number". http://staff.science.uva.nl/~johan/Muenster.pdf