The Coinductive Approach to Verifying Cryptographic Protocols

Jesse Hughes

joint work with **Martijn Warnier**

jesseh@cs.kun.nl

University of Nijmegen

Outline

I. Cryptographic protocols in general

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II. An example protocol

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III. Coalgebra primer

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IV. Temporal operators/Galois algebras

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VII. Security protocols revisited

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VII. Security protocols revisited
VIII. Paulson’s inductive method

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Part One: The Background

Cryptographic Protocols

Cryptographic Protocols, Abstract representation for:

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- Distributing secret keys over an open (insecure) network.

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Cryptographic Protocols, Abstract representation for:

- Distributing secret keys over an open (insecure) network.
- Authenticating principals to each other.
- Assuring secrecy of message content.
- Assuring integrity of messages.
- A combination of all of the above.

An example Protocol

Bilateral Key Exchange with Public Key Protocol: a simple protocol for distributing a symmetric key.

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Principal **B** sends to Principal **A** a message containing

- his name, B,

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Principal **B** sends to Principal **A** a message containing

- his name, B,
- and a nonce, N_B.

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Principal **B** sends to Principal **A** a message containing

- his name, B,
- and a nonce, N_B.

This is (mostly) encrypted with **A**'s public key, pk_A.

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A replies with a message containing

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A replies with a message containing

- a hash of **B**'s nonce,
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All of this is encrypted with **B**'s public key, pk_B.

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3. $\mathbf{B} \to \mathbf{A} :\quad \{\mathrm{Sha}(N_A)\}_{K_{AB}}$

B replies with a hash of N_A encrypted with the session key K_{AB}.

Analysis

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We analyze the protocol using a Dolev-Yao security model.
That is, we create a model consisting of

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We then prove that the conditions above hold.

Analysis

Requirements:

needed for:

- to model abstract data types messages

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• to model abstract data types	messages
• to model dynamic systems	users' knowledge

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The language CCSL allows all of this.

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CCSL is built upon an abstract mathematical foundation.

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Requirements:

	theory:
• to model abstract data types	algebra
• to model dynamic systems	coalgebra
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Analysis

Requirements:

	theory:
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• to model dynamic systems	coalgebra
• to use temporal reasoning	Galois algebra

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Part Two: The Theory

Algebra primer

Let Σ be a signature, i.e.,

$$\Sigma = \{f_i^{(n_i)} \mid i \in I\}.$$

Algebra primer

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A Σ-algebra is a set A together with an interpretation for each f_i.

Algebra primer

Example: $\Sigma = \{e, -^{-1}, \times\}$.

$$\begin{tikzcd}
1 \arrow[dr, "e"'] & A \arrow[d, "-^{-1}" description] & A \times A \arrow[dl, "\times"] \\
 & A &
\end{tikzcd}$$

Algebra primer

Example: $\Sigma = \{e, -^{-1}, \times\}$.

$$\begin{array}{c} 1 \; + \; A \; + \; A \times A \\ \downarrow \\ A \end{array}$$

Algebra primer

Example: $\Sigma = \{e, -^{-1}, \times\}$.

$$
\begin{array}{ccc}
1 + A + A \times A & \qquad & FA \\
\downarrow & & \downarrow \\
A & & A
\end{array}
$$

Let F:**SET**$\rightarrow$**SET** be given. An F-algebra is a set A with a structure

$$
\begin{array}{c}
FA \\
\downarrow \\
A
\end{array}
$$

Algebra primer

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\end{array}
$$

For polynomial functors, an F-algebra is a universal algebra.

Coalgebra primer

Example:

$$
\begin{array}{ccc}
1 + A + A \times A & \qquad & FA \\
\downarrow & & \downarrow \\
A & & A
\end{array}
$$

Coalgebra primer

Example:

$$\begin{tikzcd}[column sep=small] 1 + & A \arrow[from=2-2, u] & + A \times A & & FA \\ & A & & & A \arrow[u] \end{tikzcd}$$

An F-coalgebra is a set A with a structure

$$\begin{tikzcd} FA \\ A \arrow[u] \end{tikzcd}$$

Coalgebra primer

Example:

$$\begin{array}{ccc} 1 + A + A \times A & \qquad & FA \\ \uparrow & & \uparrow \\ A & & A \end{array}$$

An F-coalgebra is a set A with a structure

$$\begin{array}{c} FA \\ \uparrow \\ A \end{array}$$

Think: a coalgebra is a set in which each element can be decomposed as elements of a structured set.

Coalgebra primer

Example:

$$\begin{array}{ccc} 1 + A + A \times A & \qquad & FA \\ \uparrow & & \uparrow \\ A & & A \end{array}$$

Coalgebras model non-well-founded structures, including infinitary trees, streams, etc.

Coalgebra primer

Example:

$$
\begin{array}{ccc}
1 + A + A \times A & \qquad & FA \\
\uparrow & & \uparrow \\
A & & A
\end{array}
$$

Coalgebras can also represent dynamic systems.

Coalgebra primer

Example:

$$
\begin{tikzcd}
1 + A + A \times A & & F A \\
A \arrow[u] & & A \arrow[u]
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$$

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In security protocols, the principals' knowledge changes over time as messages are sent and received.

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Coalgebras can also represent dynamic systems.

In security protocols, the principals' knowledge changes over time as messages are sent and received.

Hence, we use a coalgebraic model.

Coalgebraic signatures

An algebraic signature is given by declarations:

$$f_i : X^{n_i} \longrightarrow X$$

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Equivalently,

$$f : X \longrightarrow \prod_i F_i X$$

Examples

FX	Initial algebra	Final coalgebra
$Z \times X$	$\emptyset$	infinite streams

Examples

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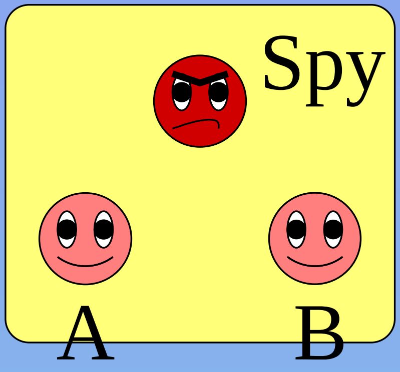
$1 + Z \times X$	finite streams	finite and infinite streams
$1 + X \times X$	finite trees	finite and infinite trees

Examples

FX	Initial algebra	Final coalgebra
$Z \times X$	$\emptyset$	infinite streams
$1 + Z \times X$	finite streams	finite and infinite streams
$1 + X \times X$	finite trees	finite and infinite trees
$\mathcal{P}_\omega X$	finite, arb. branching trees	Kripke frame

Our coalgebra



Consider a run with three principals: **A**, **B** and the **Spy**.

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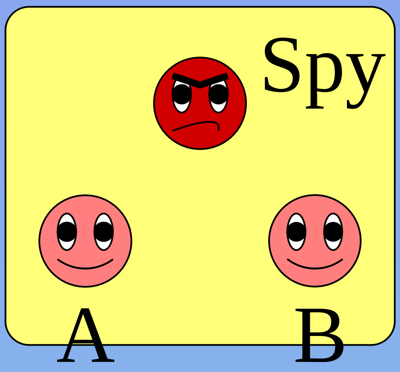
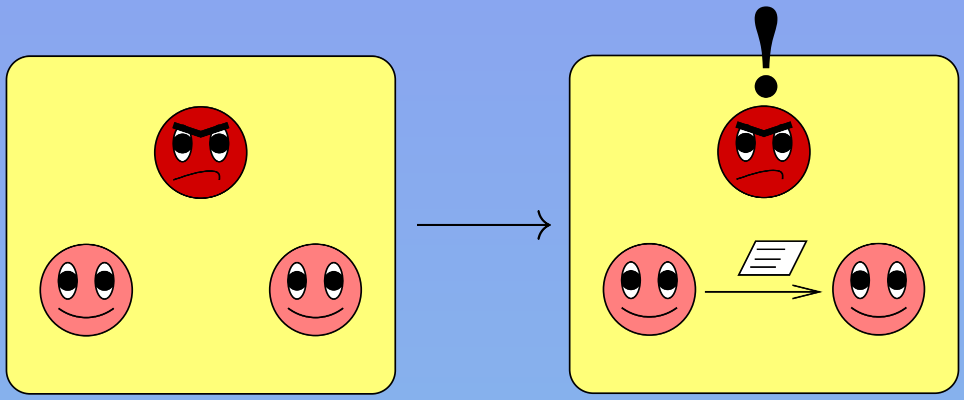


Consider a run with three principals: **A**, **B** and the **Spy**.

Suppose that **A** sends a message to **B**.

Our coalgebra



Then, in the next instant, the **Spy** learns the message.

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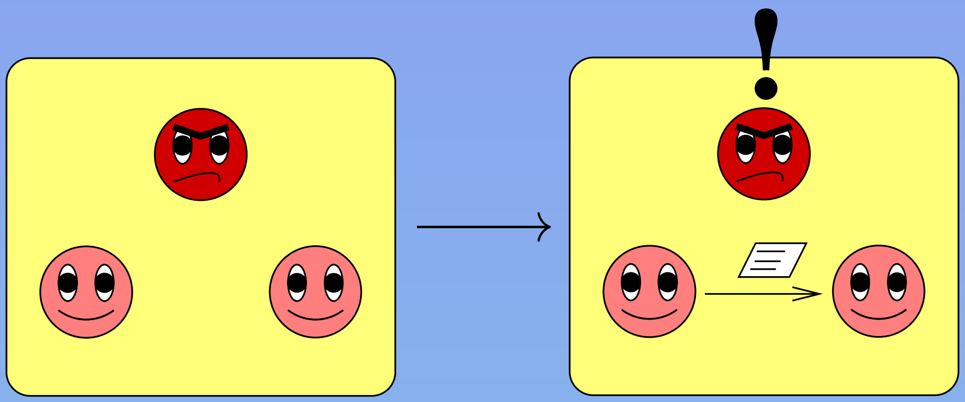
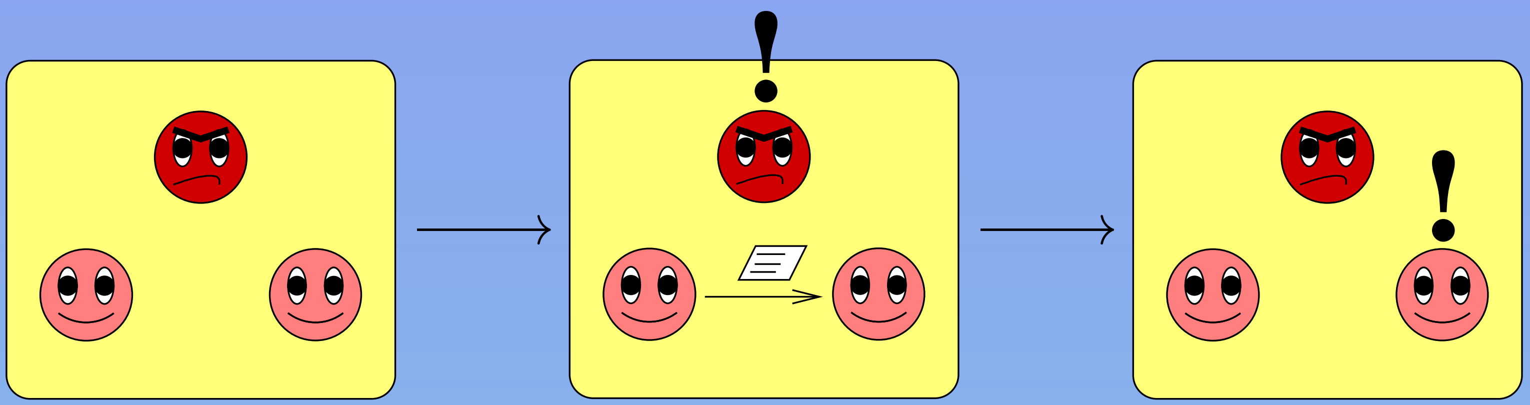


Then, in the next instant, the **Spy** learns the message.

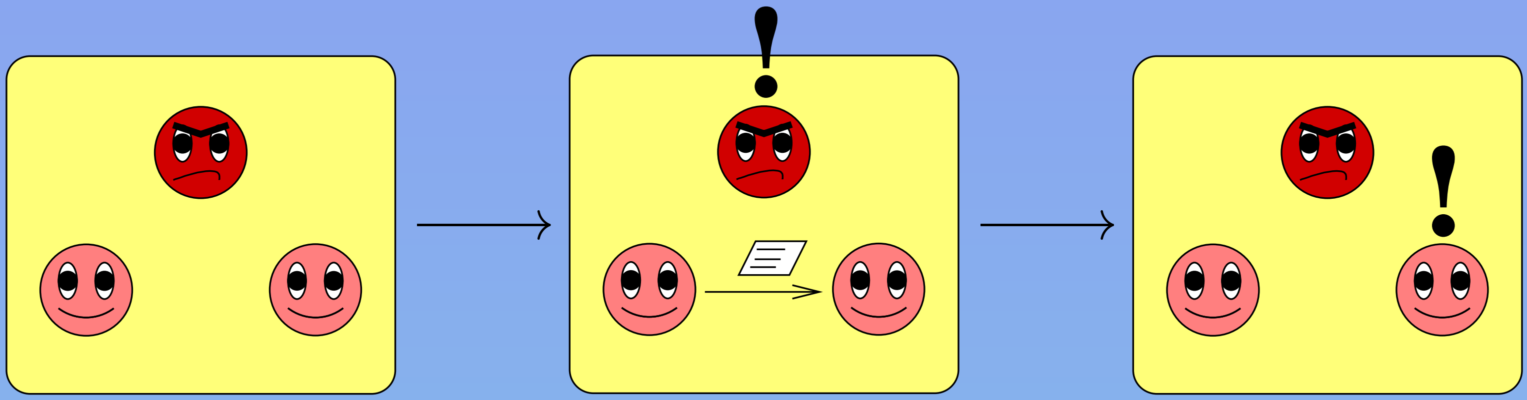
Supposing that the message arrives at that time, then…

Our coalgebra



…the next instant, **B** learns the message, too.

Our coalgebra



So, to describe this system, we use a coalgebra with

- a method giving the next state,

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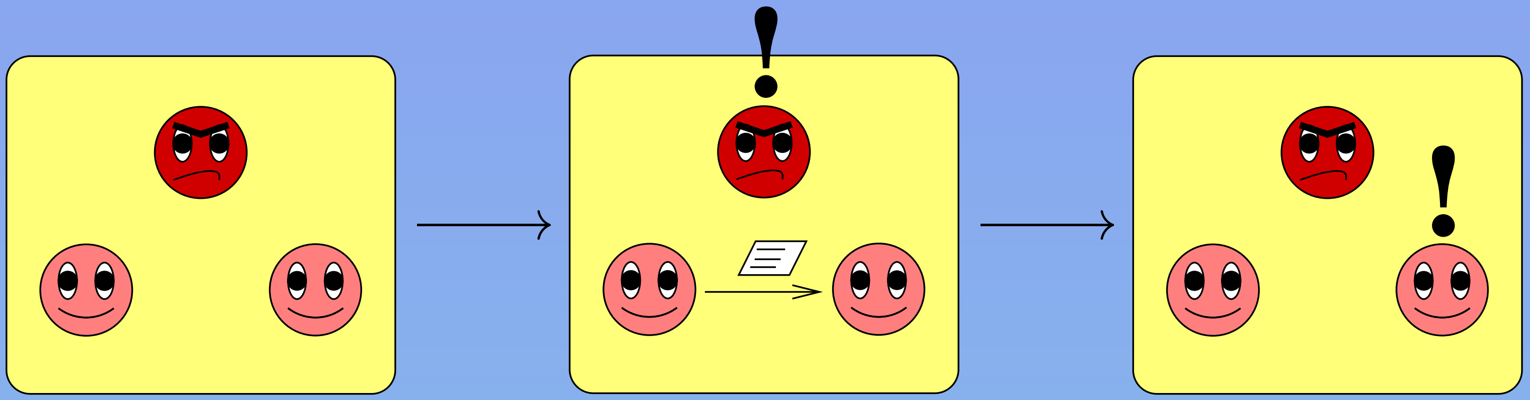

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- attributes describing the action occurring,

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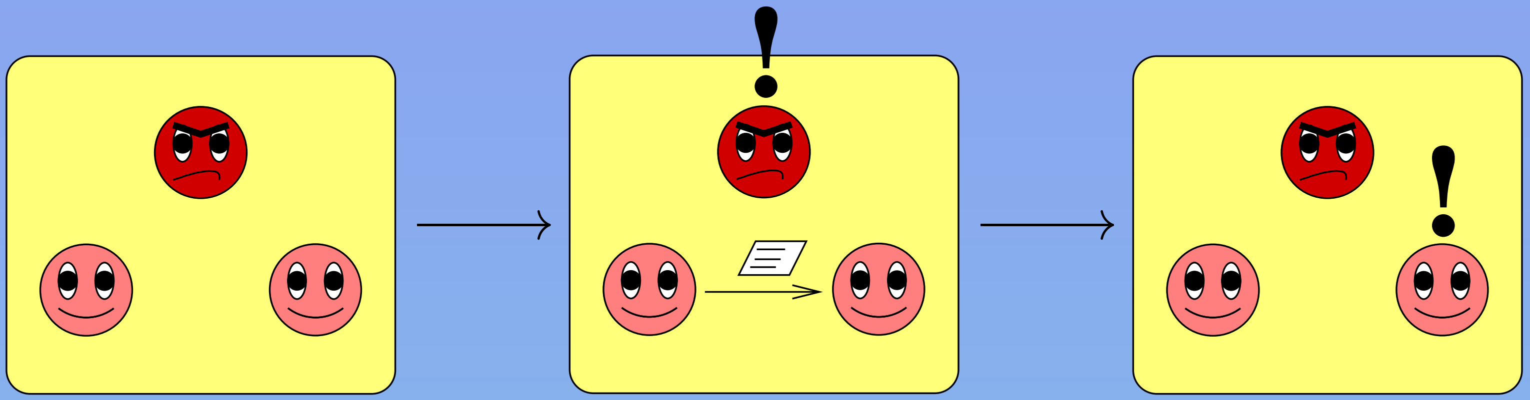
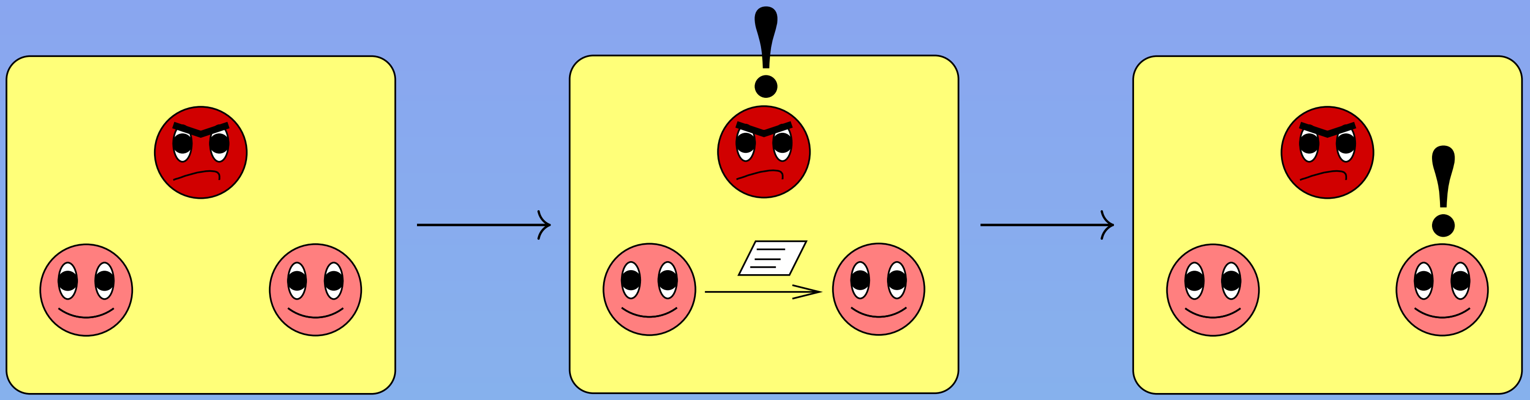


So, to describe this system, we use a coalgebra with

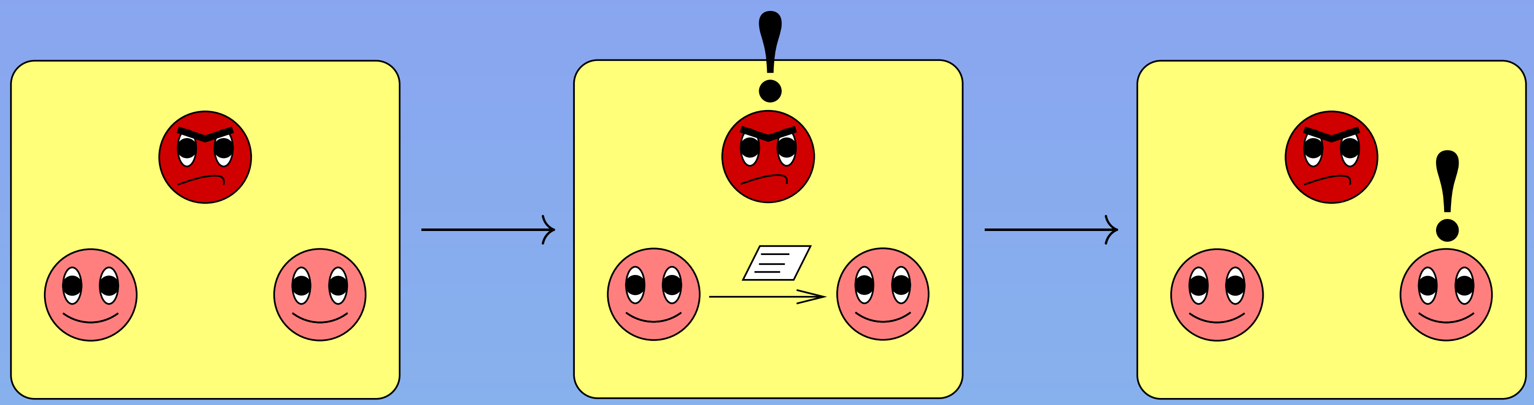
- a method giving the next state,
- attributes describing the action occurring,
- attributes describing the participants' knowledge.

Our coalgebra



$$
\begin{array}{l}
\mathsf{MsgContext} : \textbf{CLASSSPEC} \\
\quad \textbf{METHOD} \\
\qquad \texttt{next} \colon \mathsf{Self} \longrightarrow \mathsf{Self} \\
\qquad \texttt{action} \colon \mathsf{Self} \longrightarrow \{\textbf{idle}, \textbf{sent}, \textbf{received}\} \\
\qquad \texttt{knows} \colon \mathsf{Self} \times \mathsf{Princ} \longrightarrow [\mathsf{Message} \longrightarrow \mathsf{Bool}]
\end{array}
$$

Our coalgebra



We would like to prove, e.g., that

*The **Spy** never learns the session key.*

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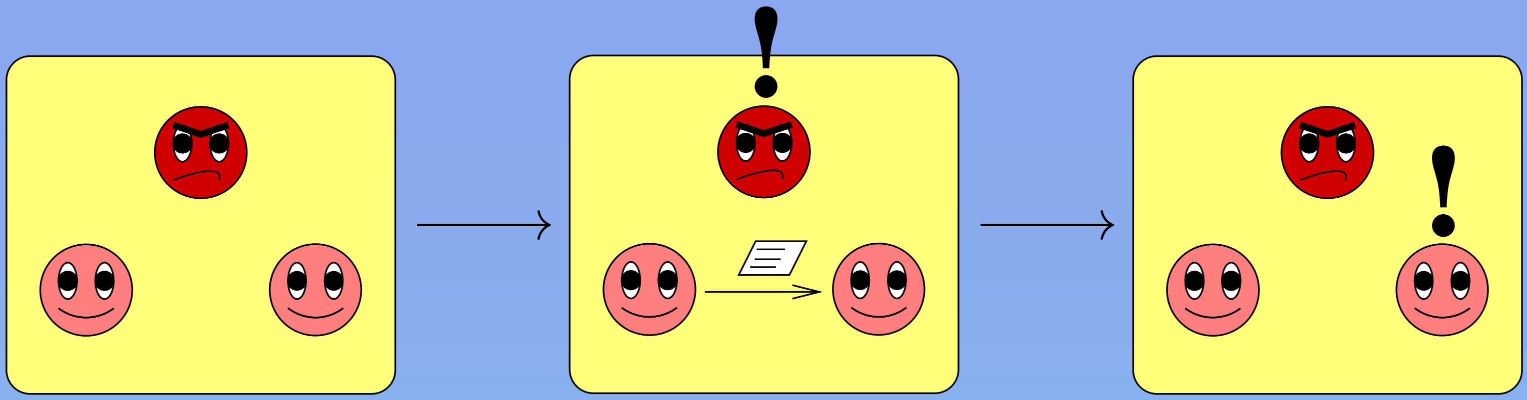

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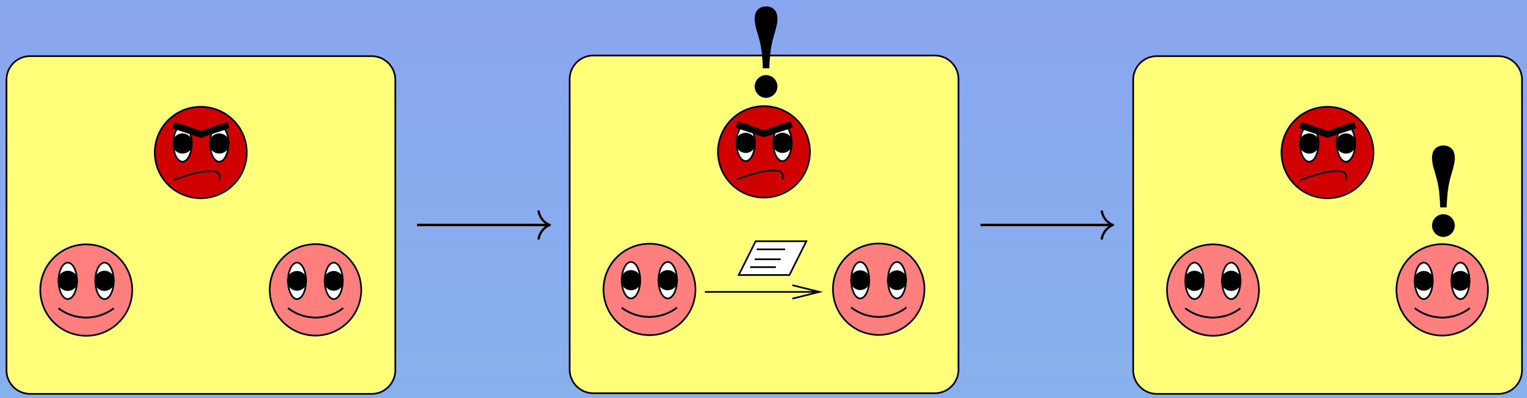

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Categories of coalgebras come with temporal operators, which we can understand in terms of Galois algebras.

Galois algebras

A Galois algebra is a complete, Boolean algebra $\mathbb{P}$ together with an operation

$$[\,] : \mathbb{P} \longrightarrow \mathbb{P}$$

which preserves meets.

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Think: $[\,]P(x)$ means P holds for all successor states of x.

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With just these assumptions, we can develop a remarkable amount of temporal logic.

Galois algebras

$$\langle\ \rangle^{\leftarrow} \quad \dashv \quad [\ ]$$

$[\ ]$ is part of a Galois connection, with left adjoint $\langle\ \rangle^{\leftarrow}$.

Galois algebras

$$\begin{array}{ccc} \langle\ \rangle^{\leftarrow} & \dashv & [\ ] \\ [\ ]^{\leftarrow} & & \langle\ \rangle \end{array}$$

Each operator has a conjugate,

$$[\ ]^{\leftarrow} = \neg\langle\ \rangle^{\leftarrow}\neg$$
$$\langle\ \rangle = \neg[\ ]\neg$$

Galois algebras

$$\langle\;\rangle^{\leftarrow} \quad \dashv \quad [\;]$$

$$[\;]^{\leftarrow} \quad \vdash \quad \langle\;\rangle$$

This yields another Galois connection.

Galois algebras

$$\langle\,\rangle^{\leftarrow} \quad \dashv \quad [\,] \qquad \text{Next time}$$

$$[\,]^{\leftarrow} \quad \vdash \quad \langle\,\rangle$$

In our interpretation, $[\,]$ means "in every next state".

$$[\,]P = \{p \mid \forall p \to r \,.\, P(r)\}$$

Galois algebras

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In our interpretation, $[\;]$ means “in every next state”.

$$[\;]P = \{p \mid \forall p \to r \,.\, P(r)\}$$

A proposition P such that P implies $[\;]P$ is called an $invariant$.

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A proposition P such that P implies $[\,]P$ is called an $invariant$.
Invariants are the coalgebraic analogues to inductive predicates.

Galois algebras

Some time preceding	$\langle\,\rangle^{\leftarrow}$	$\dashv$	$[\,]$	Next time
Always preceding	$[\,]^{\leftarrow}$	$\vdash$	$\langle\,\rangle$	Some next time

This induces the remaining interpretations.

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This allows us to represent statements like

> *If **B** receives a message at time t, then **B** knows the message at $t+1$.*

Galois algebras

Some time preceding	$\overleftarrow{\langle\ \rangle}$	$\dashv$	$[\ ]$	Next time
Always preceding	$\overleftarrow{[\ ]}$	$\vdash$	$\langle\ \rangle$	Some next time

Note: from just a complete partial order with a meet-preserving operator, we get the remaining three operators.

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But wait! There's more…

Fixed point operators

$\Box$ Always

We can define an "always" operator via a fixed point construction:

$$\Box P = \nu Z \,.\, P \wedge [\,]Z$$

Fixed point operators

$\square$ Always

We can define an "always" operator via a fixed point construction:

$$\square P = \nu Z \,.\, P \wedge [\,]Z$$

$\square P$ is the greatest invariant contained in P.

Fixed point operators

$\Box$ Always

We can define an "always" operator via a fixed point construction:

$$\Box P = \nu Z \,.\, P \wedge [\,]Z$$

$\Box P$ is the greatest invariant contained in P.

This operator preserves meets, so we have *another* Galois algebra.

Fixed point operators

$$\text{Once} \quad \overleftarrow{\Diamond} \quad \dashv \quad \Box \quad \text{Always}$$

$$\text{Previously} \quad \overleftarrow{\Box} \quad \vdash \quad \Diamond \quad \text{Eventually}$$

This yields the remaining operators and interpretations.

Fixed point operators

Once	$\overleftarrow{\Diamond}$	$\dashv$	$\Box$	Always
Previously	$\overleftarrow{\Box}$	$\vdash$	$\Diamond$	Eventually

Now, we can represent statements like

> *The **Spy** never learns the private keys of the other principals.*

Fixed point operators

Once	$\overleftarrow{\Diamond}$	$\dashv$	$\Box$	Always
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All of this structure just comes from the presence of the "next time" operator, $[\,]$.

Part Three: CCSL

Overview

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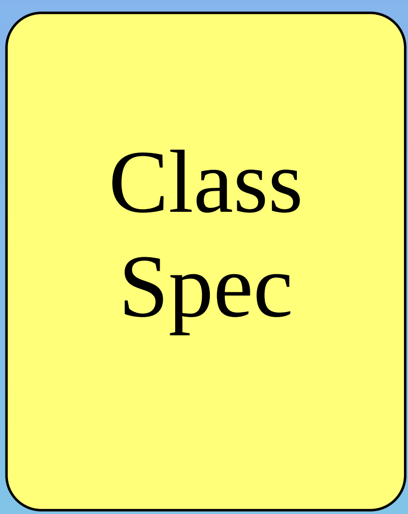
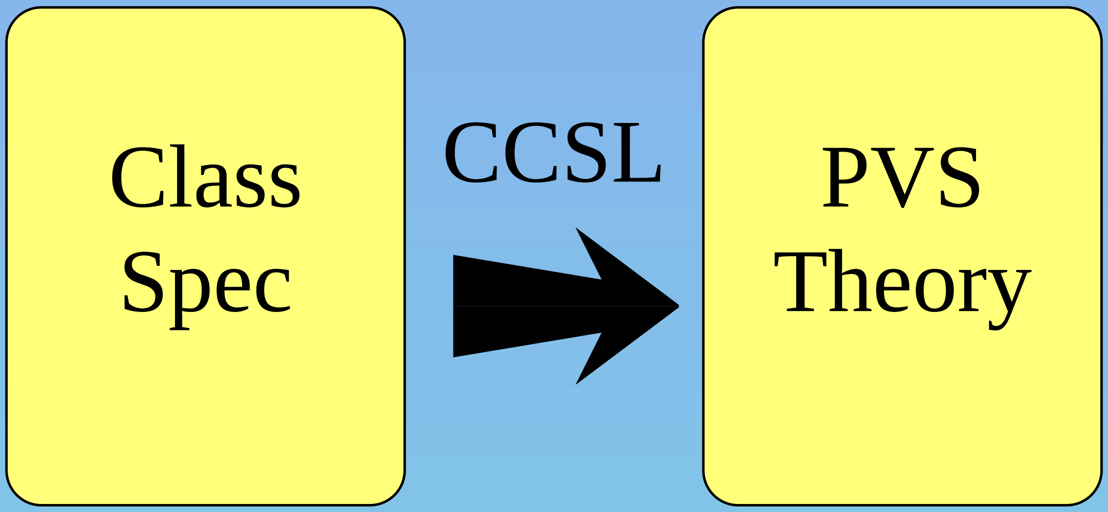
CCSL provides a means for expressing a class specification in terms of these theories.



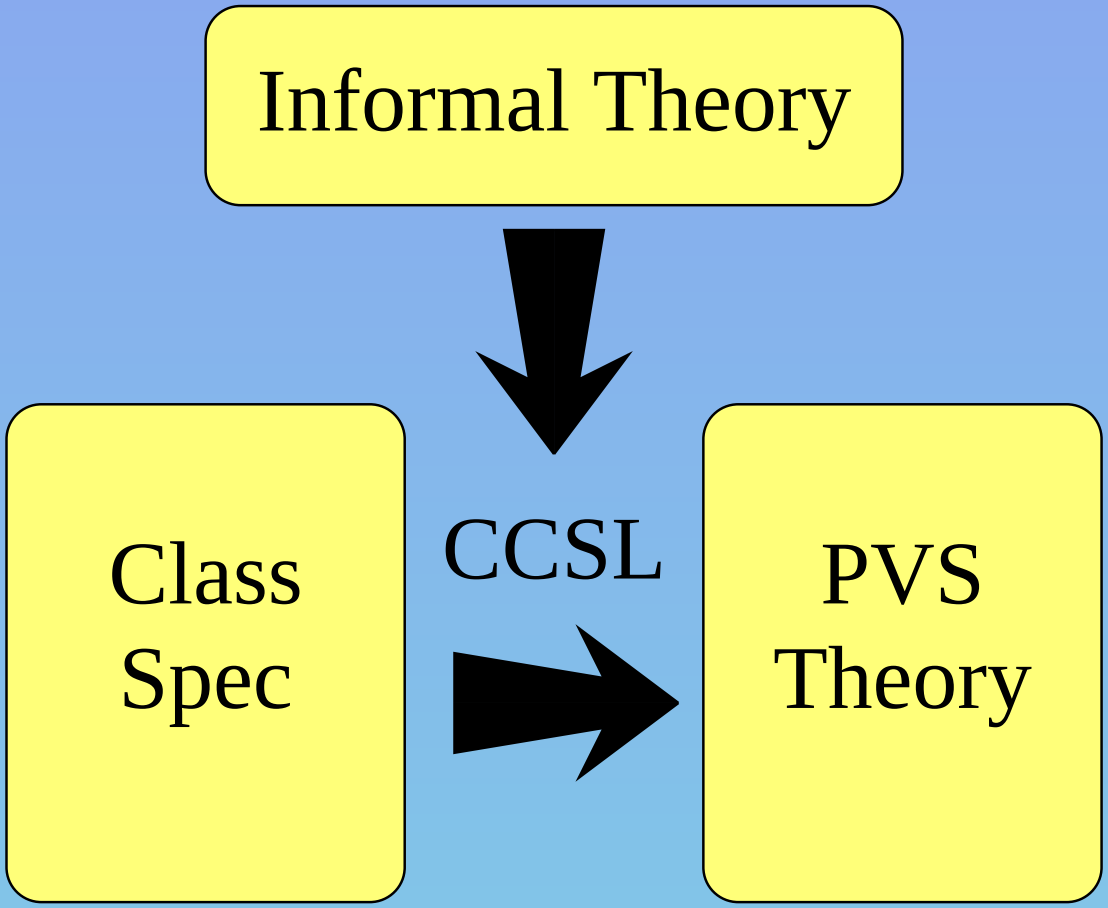
Overview

The compiler translates a specification into a formal, logical theory (in PVS/Isabelle).



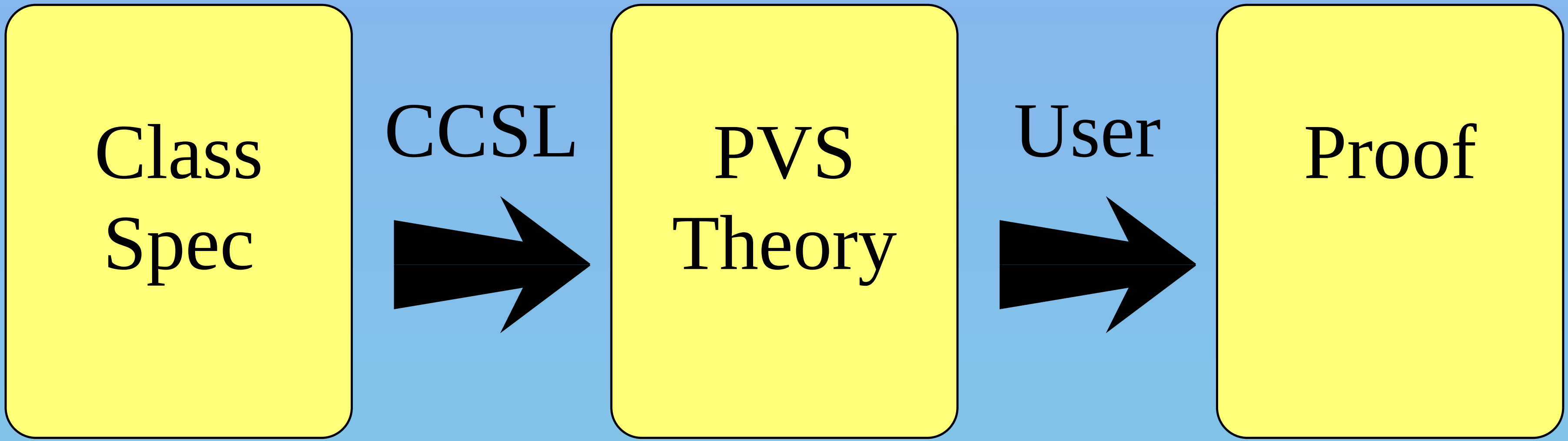
Overview

This theory includes induction (algebra), coinduction (coalgebra), temporal axioms (Galois algebra), etc.



Overview

The user then proves the correctness of the specification in the theorem prover.



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	Models
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- dynamic structure by a class (e.g. principal's current knowledge).

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A class specification consists of:

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Here's where the assumptions come in!

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Correctness conditions for a specification are represented as theorems.

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Input: class and abstract data specifications.

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Part Four: The Application

Back to security protocols

Considering protocols like:

$$\begin{array}{lll} 1. & \mathbf{B} \rightarrow \mathbf{A}: & B, \{N_B, B\}_{\mathrm{pk}_A} \\ 2. & \mathbf{A} \rightarrow \mathbf{B}: & \{\mathrm{Sha}(N_B), N_A, A, K_{AB}\}_{\mathrm{pk}_B} \\ 3. & \mathbf{B} \rightarrow \mathbf{A}: & \{\mathrm{Sha}(N_A)\}_{K_{AB}} \end{array}$$

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- Perfect cryptography assumption
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- Other assumptions: freshness, "perfect" hashes, true randomness of nonces.

Back to security protocols

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MsgContext: sample methods

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The basic methods represent

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Knowledge does not change if idle.

MsgContext: sample theorem

$\mathsf{MsgContext} : \textbf{CLASSSPEC}$

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$\texttt{next} \colon \mathsf{Self} \longrightarrow \mathsf{Self}$

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THEOREM

$$\begin{aligned} & \forall (P : \mathsf{Princ}, m : \mathsf{Message}) : \\ & \qquad x.\texttt{knows}(P)(m) \Rightarrow \\ & \qquad\quad x.\texttt{next}.\texttt{knows}(P)(m). \end{aligned}$$

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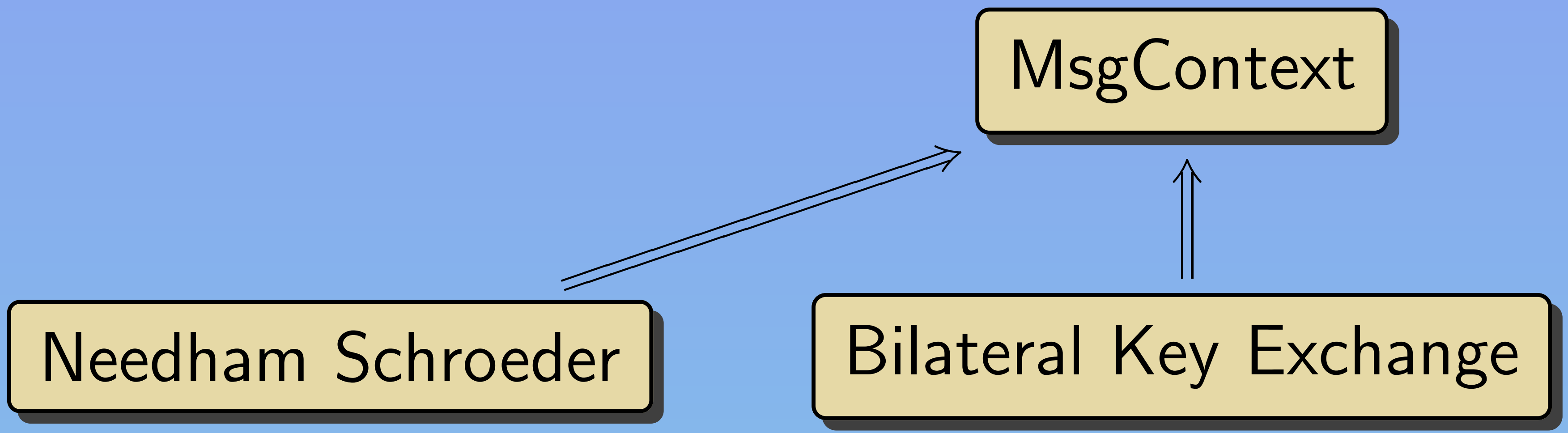

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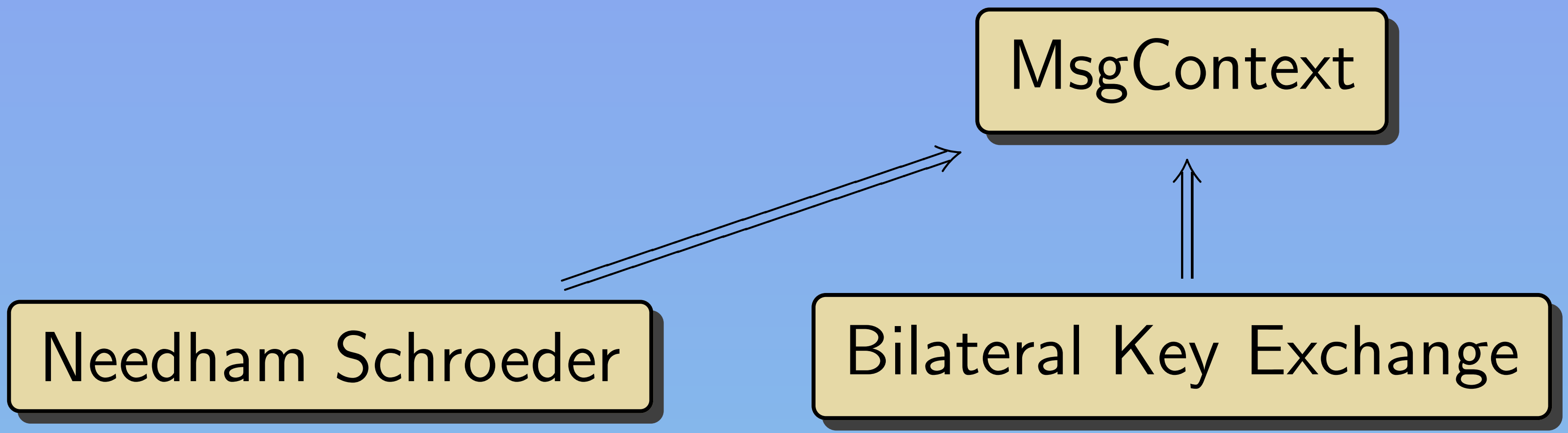

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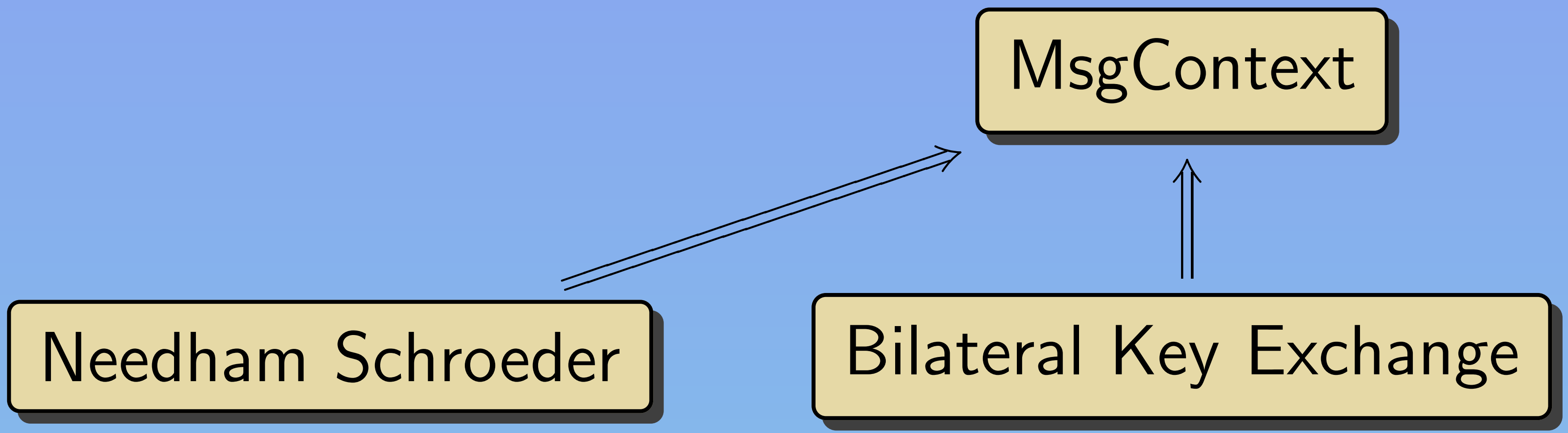

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All of this is easily expressible in CCSL, using our MsgContext protocol.

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Admittedly, proving it is not so easy.

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He considers the set of finite traces for a protocol. This set can be given by a least fixed point construction, i.e., by an initial algebra.

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The main theoretical difference is that we consider infinite traces as models, while Paulson considers finite traces.

Comparison

There are a number of practical differences in Paulson's work and our own.

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As well, our specification places fewer restrictions on the behavior of the participants but we pay for this generality!

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More on CCSL can be found here:
`http://wwwtcs.inf.tu-dresden.de/~tews/ccsl/`