A Semantics for Means-End Ascriptions

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Outline

1. Means and ends, informally

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   - Norms in Knowledge

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   - Propositional Dynamic Logic
   - Conditional means-end ascriptions

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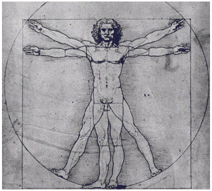
Introduction to *Norms in Knowledge*

An epistemological investigation.



Epistemology:

- Knowledge of descriptive claims

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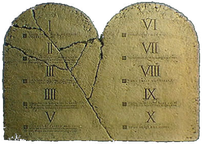

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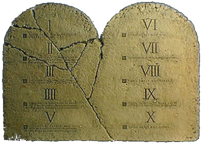

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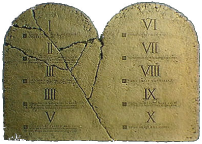

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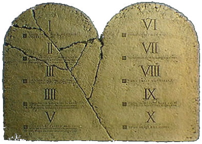

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Applied to technical artifacts:

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Applied to technical artifacts:

- Knowledge of normative claims
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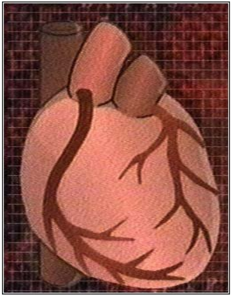
    - Prescriptive — ought to do
      Artifacts: HOWTOs
    - Functional — things ought to do
      Artifacts: artifactual functions

Some examples of functional ascriptions



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- “That switch mutes the television.”

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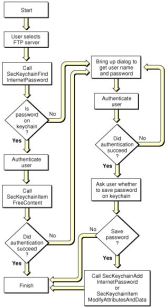

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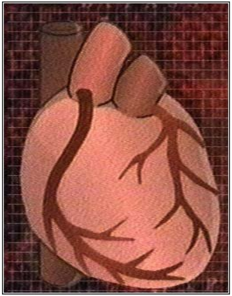

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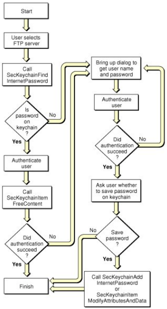

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How functions relate to means and ends

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One can use the switch to mute the television.

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There is an action involving the switch that will cause the television to be muted.

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- Functions imply means-end relations.

How functions relate to means and ends

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There is an action involving the switch that will cause the television to be muted.

- Functions imply means-end relations.
- Step one: Provide a semantics for means-end relations.

What is an end? a mean?

- An end is some desirable condition.

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- A means is a way of making the end true.

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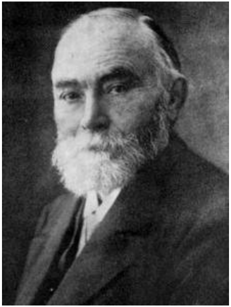
- Ends-in-themselves?
- Objects as means?

Possible worlds and making propositions come true

Ends are propositions we want to make true.

Possible worlds and making propositions come true



Frege

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But actions don't change the meaning of propositions.

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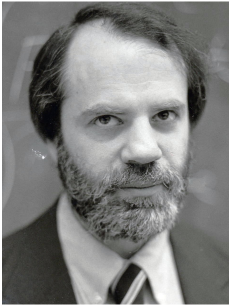

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Think of a set of possible worlds.

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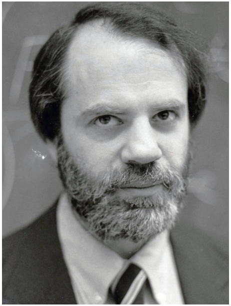

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At each time, *one* world is the actual world.

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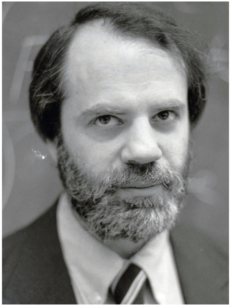

Kripke

Ends are propositions we want to make true.

But actions don't change the meaning of propositions.

Think of a set of possible worlds.

At each time, *one* world is the actual world.

And at each world, every proposition is true or false.

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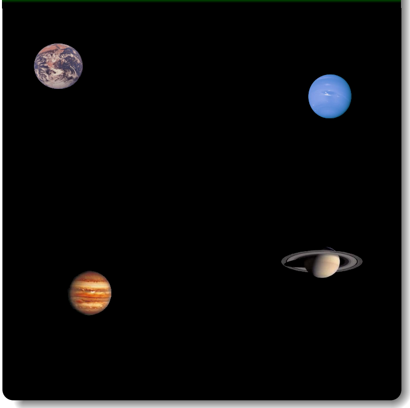
A simple example of possible worlds

Our Universe



A set of worlds involving
a footrace and starter pistol.

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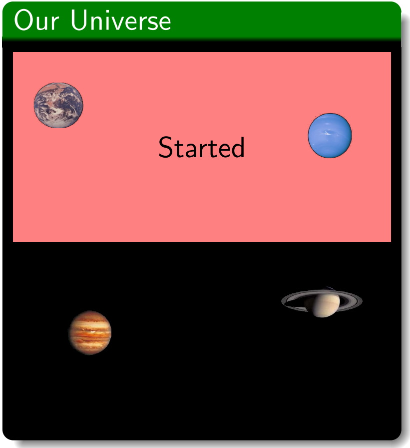

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- Two basic properties:
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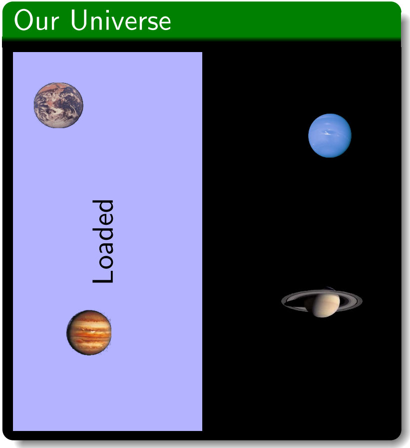

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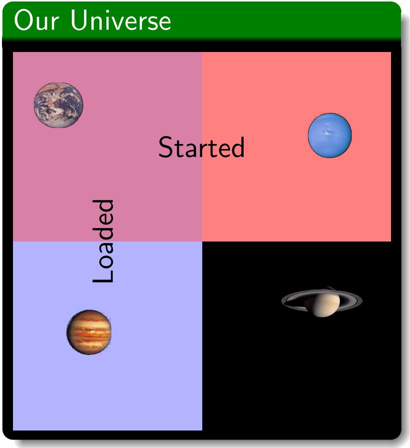

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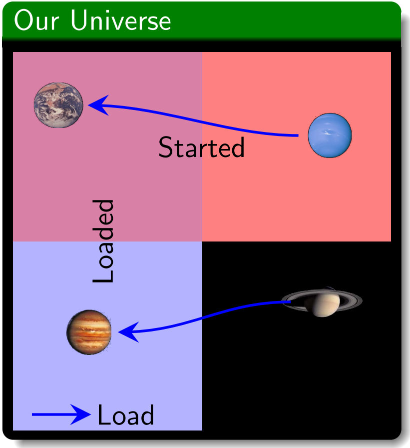

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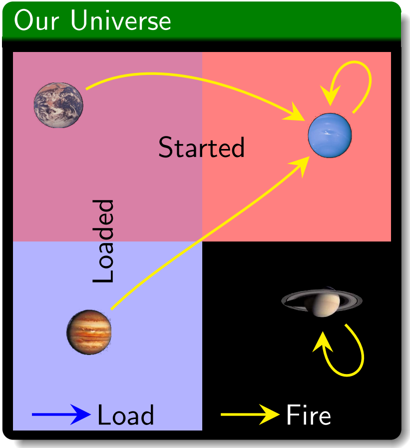

- Two basic actions:
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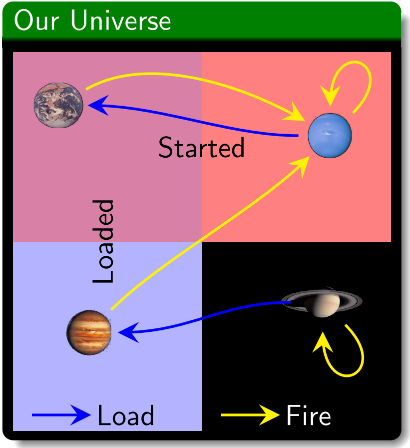

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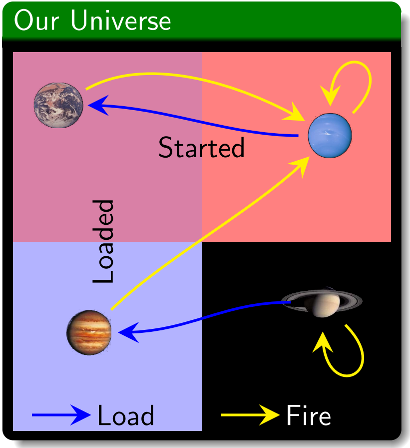

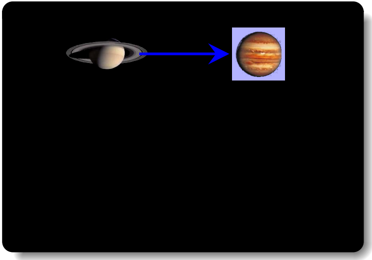

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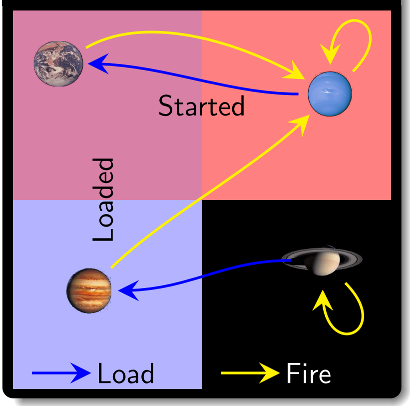

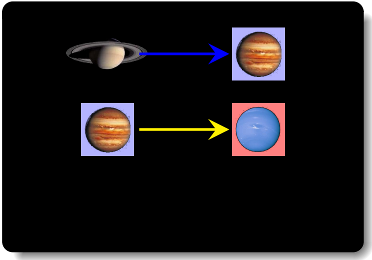

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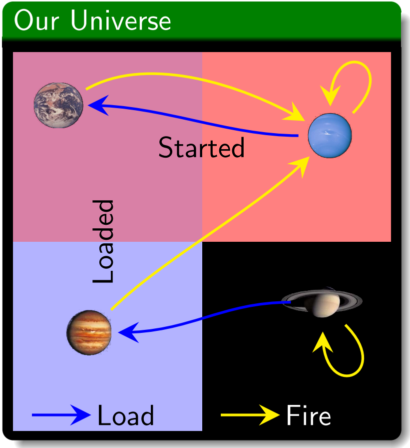

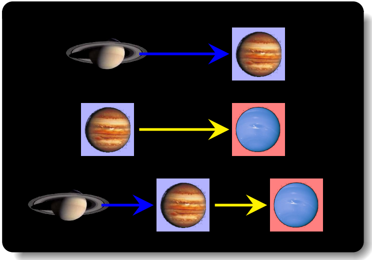

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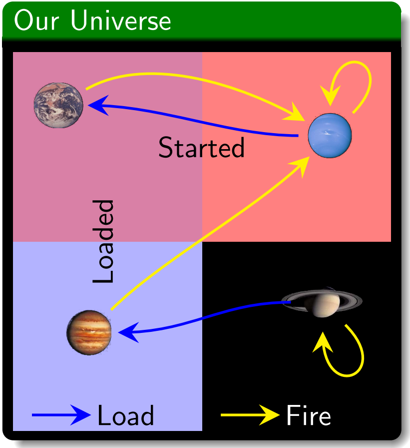

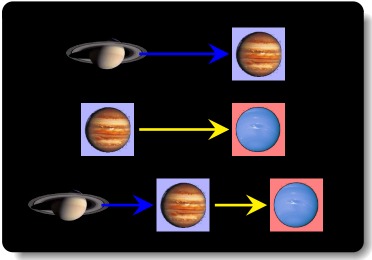

In Saturn, loading and firing is a means to starting the race.

But the world isn't deterministic, is it?

Actions may have uncertain outcomes.

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- Randomness

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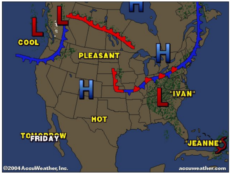

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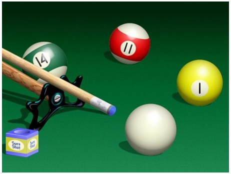

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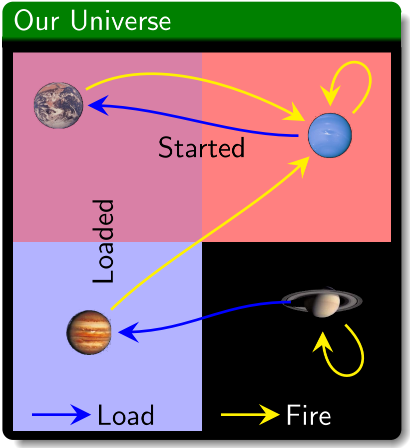
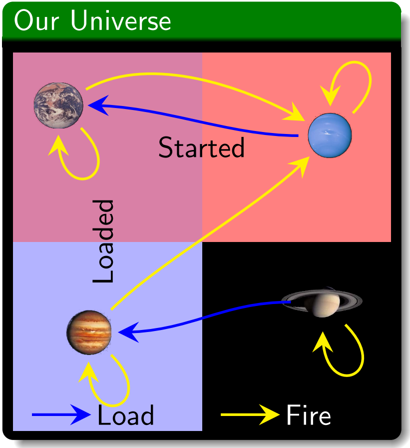
Our models should support *non-determinism.*

Adding non-determinism to our model



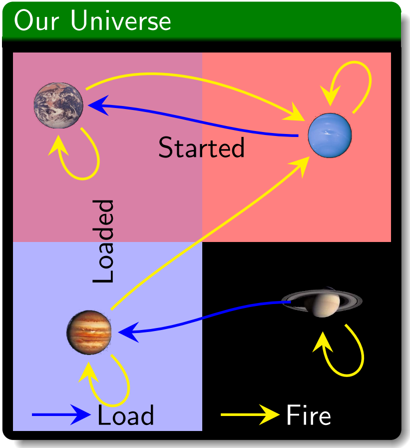
The pistol has a weak spring.

Adding non-determinism to our model



The pistol has a weak spring.
Sometimes, bullet doesn't fire, world doesn't change.

Adding non-determinism to our model



The pistol has a weak spring. Sometimes, bullet doesn't fire, world doesn't change.

In , firing the pistol results in either  or .

Introducing the formal language PDL

Propositional dynamic logic

- Extends propositional logic

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- Language comes with a set **act** of actions

Introducing the formal language PDL

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  - $[m]\varphi$—doing m *will* result in φ.

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Propositional dynamic logic

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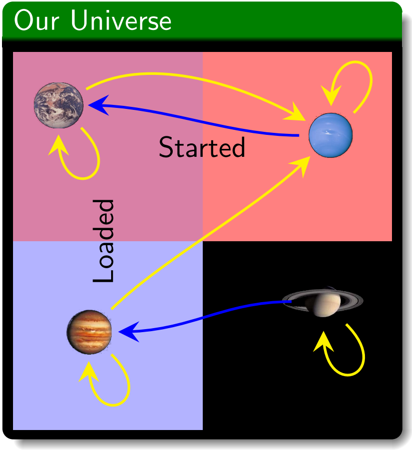
- For each action m, we add operators $[m]$, $\langle m \rangle$.
  - $[m]\varphi$—doing m *will* result in φ.
  - $\langle m \rangle\varphi$—doing m *may* result in φ.

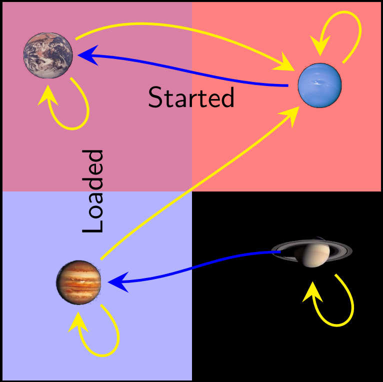
Semantics for PDL



$\mathbf{act} = \{\text{fire}, \ \text{load}\}$

Semantics for PDL

Our Universe



$\textbf{act} = \{\text{fire}, \ \text{load}\}$

$w \models [m]\varphi$

$\Updownarrow$

in w, doing m will end in φ

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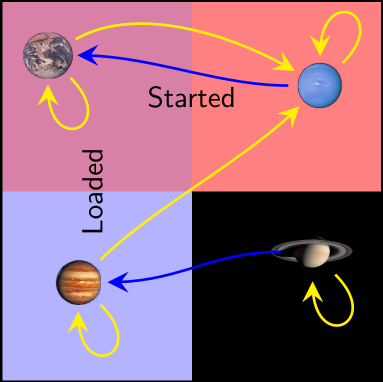

$$\mathbf{act} = \{\text{fire}, \ \text{load}\}$$

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$$w \xrightarrow{m} w' \Rightarrow w' \models \varphi$$

Semantics for PDL

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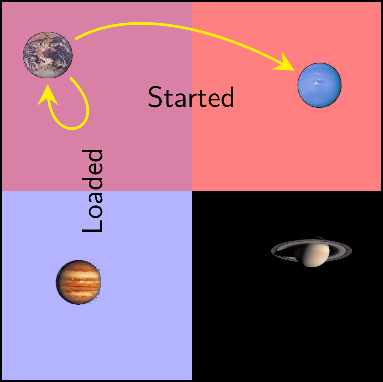

$$\textbf{act} = \{\text{fire}, \ \text{load}\}$$

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 $\models$ [fire]**Started**

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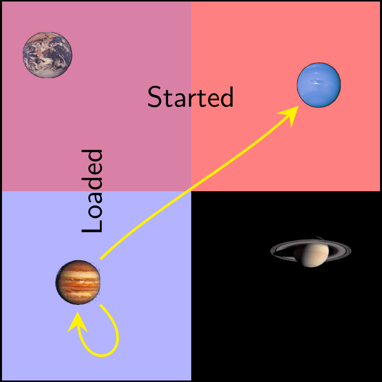

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 $\models$ [fire]**Started**

 $\not\models$ [fire]**Started**

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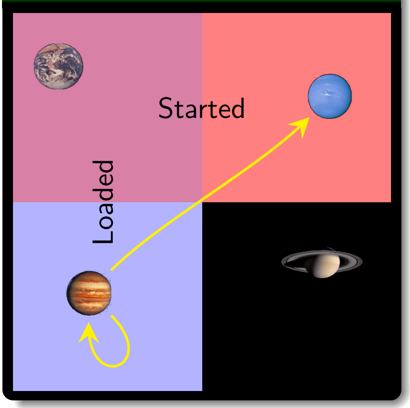
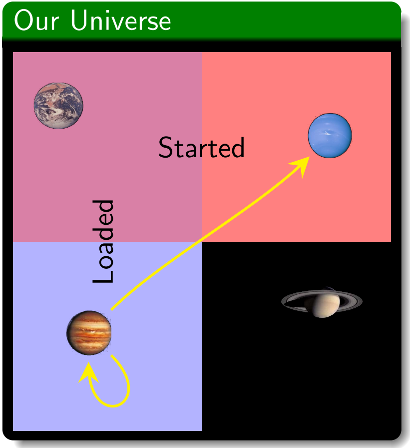


$$\textbf{act} = \{\text{fire}, \ \text{load}\}$$

$$w \models \langle m \rangle \varphi$$
$$\Updownarrow$$
in w, doing m <u>may</u> end in φ

Semantics for PDL



$$\textbf{act} = \{\text{fire}, \ \text{load}\}$$

$$w \models \langle m \rangle \varphi$$

$$\Updownarrow$$

in w, doing m <u>may</u> end in φ

$$\Updownarrow$$

$$\exists w' \, . \, w \xrightarrow{m} w' \ \& \ w' \models \varphi$$

Semantics for PDL

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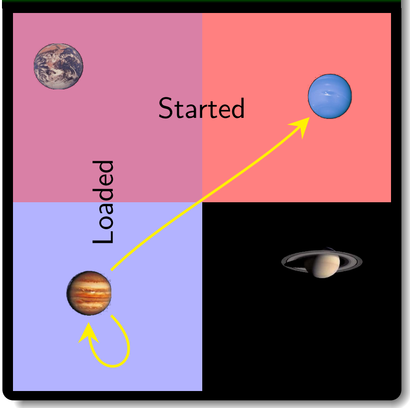

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$$w \models \langle m \rangle \varphi$$

$$\Updownarrow$$

in w, doing m may end in φ

$$\Updownarrow$$

$$\exists w' \, . \, w \xrightarrow{m} w' \; \& \; w' \models \varphi$$

 $\models \langle \text{fire} \rangle$**Started**

Means-end ascriptions in PDL

In w, m is a means to φ

Doing m in w will yield φ

Means-end ascriptions in PDL

In w, m is a means to φ

$$\underbrace{\text{Doing } m \text{ in } w \text{ \underline{will} yield } \varphi}_{w \models [m]\varphi}$$

Means-end ascriptions in PDL

In w, m is a means to φ

$$\underbrace{\text{Doing } m \text{ in } w \text{ \underline{will} yield } \varphi}$$

$$w \models [m]\varphi$$

But...
if one cannot do m, then trivially $w \models [m]\varphi$!

Means-end ascriptions in PDL

In w, m is a means to φ

$\Updownarrow$

$\underbrace{\text{Doing } m \text{ in } w \text{ will yield } \varphi}_{w \models [m]\varphi}$ and one can do m in w.

But...
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Means-end ascriptions in PDL

In w, m is a means to φ

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$\underbrace{\text{Doing } m \text{ in } w \text{ will yield } \varphi}_{w \models [m]\varphi}$ and one can do m in w.

But...
if one cannot do m, then trivially $w \models [m]\varphi$!

Aha!
$w \models \langle m \rangle$ **True**
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Means-end ascriptions in PDL

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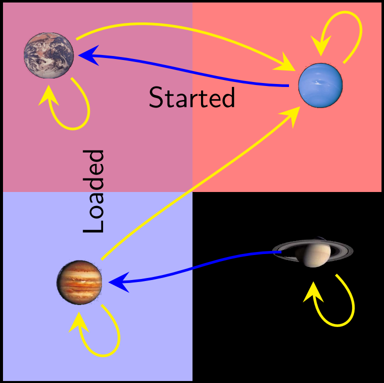
Introducing conditional means-end ascriptions

Conditional ascription:

Assuming ψ,
m is a means to φ.

Our Universe



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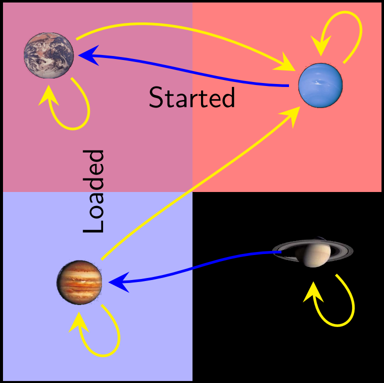

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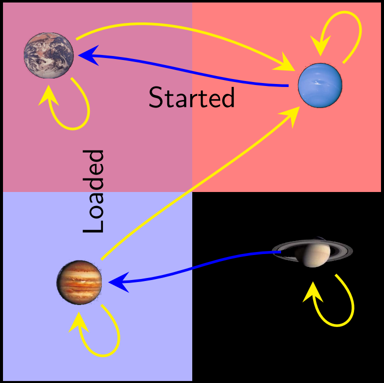

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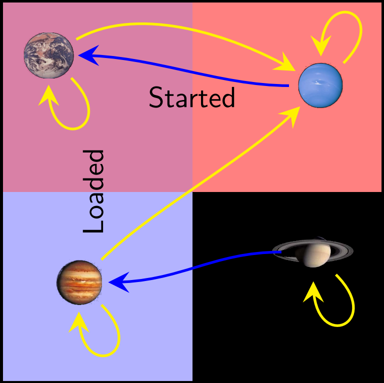

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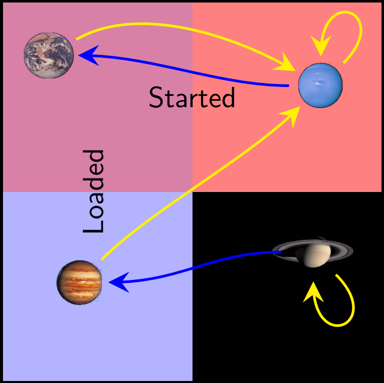

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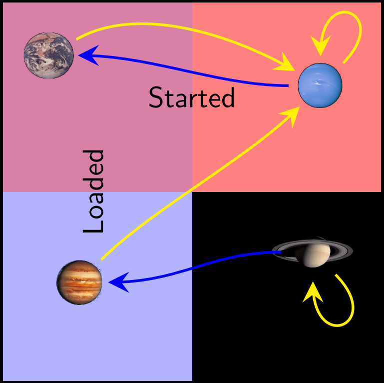

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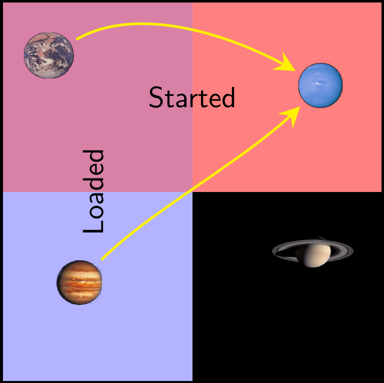
Assuming the gun is loaded, firing the gun will start the race.

Our Universe



Reevaluating material implication
(or “Why means-end reasoning is hard”)

A simple derivation:

If I had money, she would marry me.

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$[\text{rob}]\textbf{money}$

$\therefore [\text{rob}; \text{propose}]\textbf{marry}.$

Good argument:

$\textbf{Loaded} \rightarrow [\text{fire}]\textbf{Started}$

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Problem: If I rob her, she will hate me and $(\textbf{money}\ \&\ \textbf{HATE}) \not\rightarrow [\text{propose}]\textbf{marry}$.

Our conditional should be non-monotonic

Non-monotonicity

money $\rightarrow$ [propose]**marry** but
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Reasoning about means is hard.
Sometimes called the Frame Problem.

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Outline

3 Appendix
- Why use formal semantics?
- Reasoning in PDL
- Brown's Logic of Ability
- Why go fuzzy?

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  - In our project description.

A trivial bit of means-end reasoning

$\textbf{Loaded} \rightarrow [\text{fire}]\textbf{Started}$ (Given)

$[\text{load}]\textbf{Loaded}$ (Given)

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$$\models [\text{load; fire}]\textbf{Started}$$

but not $\models [\text{load; fire}]\textbf{Started} \ \& \ \langle \text{load; fire} \rangle \textbf{True}$

Ability and modal logic: Kenny's analysis

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If $S \models \varphi$, then doing S will make φ true.

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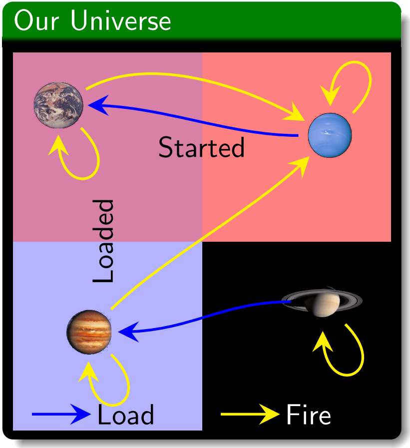
but let's not complicate matters.

Efficacy as an essential feature of means



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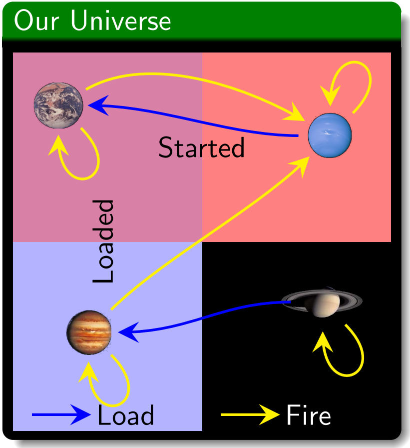

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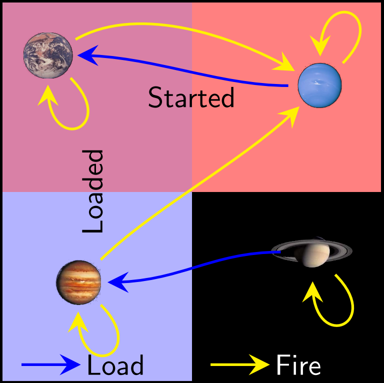

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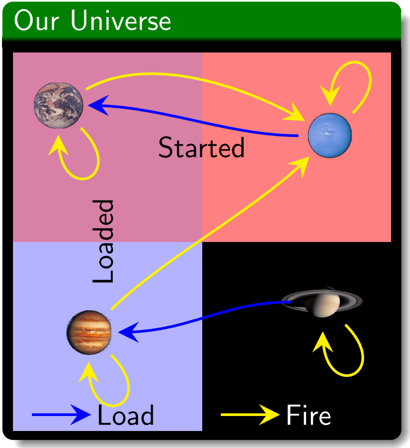

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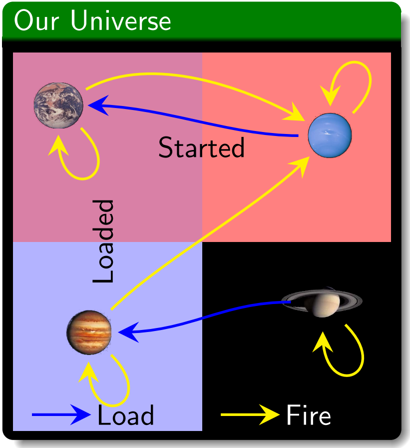
But how?

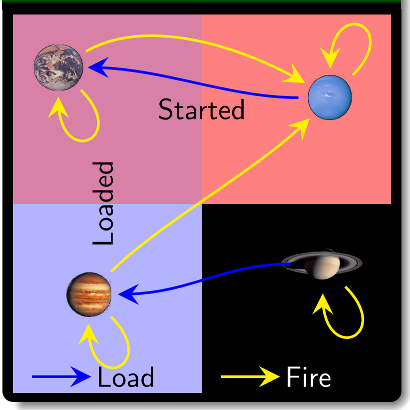
A fuzzy approach



The need for probabilities goes deeper than this.

A fuzzy approach

Our Universe



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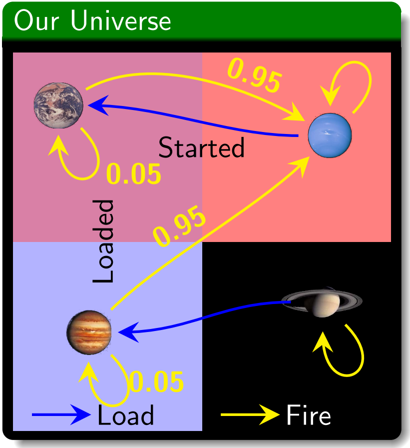
Different means to same end have different efficacies.

A fuzzy approach



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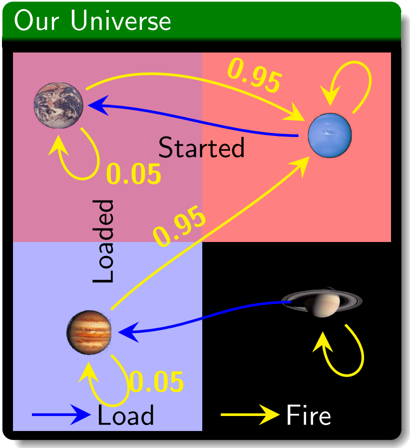
We add probabilities to our transitions. . .

A fuzzy approach



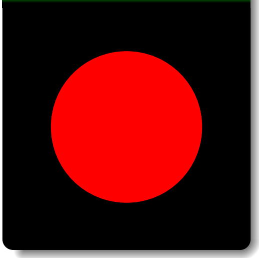
The need for probabilities goes deeper than this.

Different means to same end have different efficacies.

We add probabilities to our transitions. . .
but that's only part of the solution.

A brief introduction to fuzzy set theory

Our Universe



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A brief introduction to fuzzy set theory

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Each x is either in S or not.

A brief introduction to fuzzy set theory

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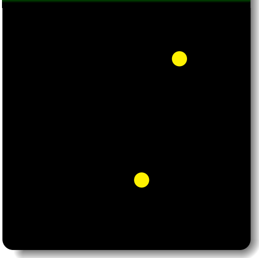
But for mere mortals...

A brief introduction to fuzzy set theory

Some propositions aren't so *crisp*.

A brief introduction to fuzzy set theory

Our Universe



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Fuzzy sets represent ambiguous propositions.

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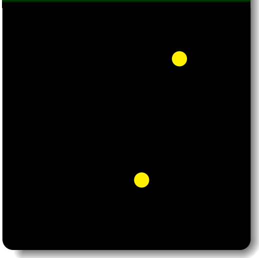
Our Universe



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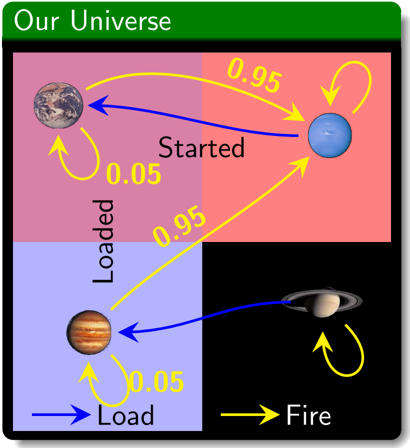
Fuzzy sets represent ambiguous propositions.

Here, $x \in S$ is assigned some value in $[0, 1]$.

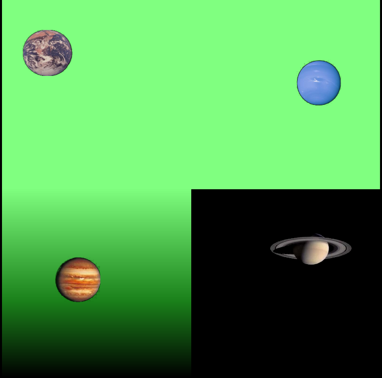
A fuzzy approach



Think again about [fire]**Started**.

A fuzzy approach

Our Universe



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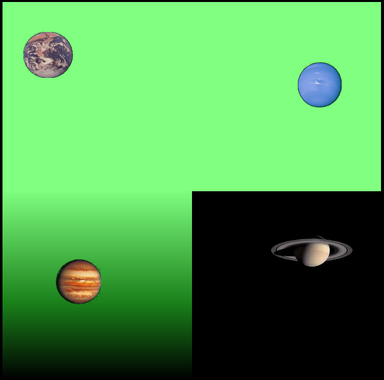
A fuzzy approach

Our Universe



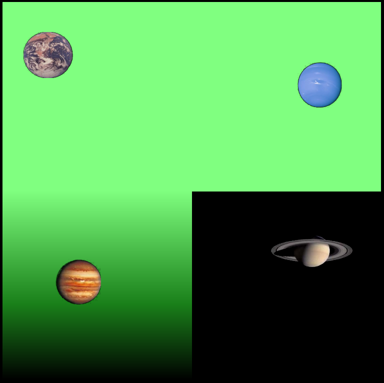
Think again about [fire]**Started**.

That is neither just true nor false.

It's a bit fuzzy.

A fuzzy approach

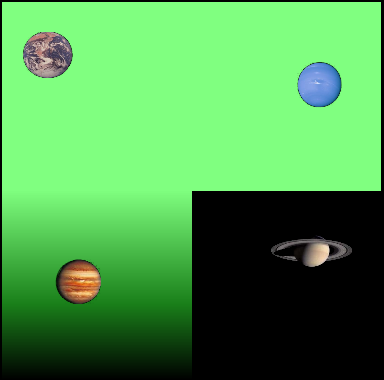
Our Universe



Now, this is a new approach.

A fuzzy approach

Our Universe



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There *are* fuzzy modal logics. . .

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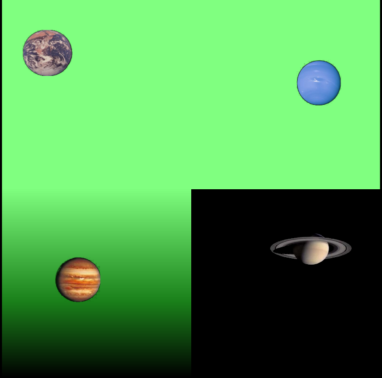
Our Universe



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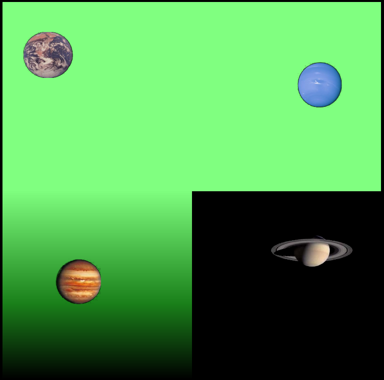
A fuzzy approach

Our Universe



Now, this is a new approach.

There *are* fuzzy modal logics. . . but they're different.

Our fuzzy dynamic logic uses expected values, not conjunctions of implications.

The Main Issues

- The relationship between ability and means.

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- The relationship between ability and means.
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- Back to artifactual functions.