Knowledge in Norms: A Sketch

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Aims

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- Require: “For $x$ to execute $U$, $x$ must know $U$.,”
Basic ingredients

Constants: Users \( (x, y, z, \ldots \in User) \)
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Artifacts

\[(x, y, z, \ldots \in User)\]

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Constants: Users $(x, y, z, \ldots \in User)$
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Relations come with types

$User \times User \times \ldots \times User \times Art \times Art \times \ldots \times Art$. 
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To \( Q \), we add a deontic operator \( \Box \), obtaining \( QD^* \).
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$$User \times User \times \ldots \times User \times Art \times Art \times \ldots \times Art.$$ 

Let $Q$ be first-order logic built on these ingredients. To $Q$, we add a deontic operator $\Box$, obtaining $QD^*$. The logic $QD^*$ includes the constant domain assumption.
Functions and plans

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But: functions are goal-directed.

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A goal is a state of affairs, a condition of the world ... a formula of Q!
Abstract user plans

We augment $QD^*$ with a new type $Plan$ (variables $U, U', \ldots$).
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$$\text{end: } Plan \longrightarrow Q$$
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$$\text{obj} : Plan \rightarrow Art$$

$$\text{end} : Plan \rightarrow Q$$

Admittedly, this type looks a bit funky.
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Each plan involves an artifact and an end.

$$\text{obj} : \text{Plan} \rightarrow \text{Art}$$
$$\text{end} : \text{Plan} \rightarrow \text{Q}$$

We could also add preconditions to a user plan, as in dynamic logic.

$$\text{pre} : \text{Plan} \rightarrow \text{Q}$$
Applications of Plans

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\[ \text{app}_w(x, U) \] is the world resulting from user \( x \) applying plan \( U \) in world \( w \).

Assumes: every user can execute every plan.
## Alternative transitions

\[ \text{app}_\cdot(-, -) : \text{World} \times \text{User} \times \text{Plan} \rightarrow ??? \]

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Note: this perspective on applications of plans is fundamentally coalgebraic!
A dynamic logic for applications

For each pair \( x \in User, U \in Plan \), we add a modal operator \([x, U]\)
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\[ w \models [x, U] \varphi \iff \text{for all } w' \in \text{app}_w(x, U), \text{ we have } w' \models \varphi. \]
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$w \models [x, U] \varphi \iff$ After $x$ applies $U$ in $w$, the formula $\varphi$ attains.
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Example:

$$[x, U] \Diamond \text{end}(U).$$
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$[x, U] \bigcirc \text{end}(U)$.

After $x$ applies $U$, the end of $U$ ought to hold.
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Example:

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Compare: \( \circ [x, U] \text{end}(U) \)
Knowledge operator?

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Introduce: \(\text{groks : User} \times \text{Plan}\).
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Introduce: $groks : User \times Plan$. 

Epistemic operators can be added as the situation requires, of course.
The total sketch

We have:

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We have:

- a basic logic $Q$ for describing the worlds;
- an extension $QD^*$ of $Q$ for ought-statements;
- an extension $QD^* + DL$ for statements involving plan execution;
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With this starting point, one can work to represent functional knowledge.
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Clearly, these tasks must go hand-in-hand.
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Concrete steps:

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Concrete steps:

- Clarify the logic $QD^* + DL$!
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  - Represent “designer”, “proper”, etc.
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- Include epistemic operator for practical reasoning?