Means-end Relations and a Measure of Efficacy

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Outline

1. Means-end relations
   - Interest I: Practical syllogisms
   - Interest II: Functional ascriptions
   - Propositional Dynamic Logic
2. Efficacy via fuzzy logic
   - Reliability as a fuzzy operator
   - The resulting fuzzy logic

Means-end relations in practical syllogisms

Practical reasoning is concerned with actions to attain desired results.

Typical practical syllogisms include premises:

- an assertion that some end φ is desirable,
- an assertion that (given ψ), the action α is related to φ,
- an assertion that ψ.

The conclusion is an *action* or an *intention*.

This premise is a *means-end relation*.

An example from von Wright

I want to make the hut habitable.
Unless I heat the hut, it will not be habitable.

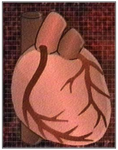
Therefore I must heat the hut.

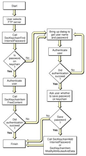
- Expression of an agent's desire,
- A *necessary* means-end relation,
- Concludes in a *necessary* action.

Note: distinct premises

But *necessary* means-end relations are a bit tricky.

Functional ascriptions





- “The function of the heart is to pump blood.”
- “That switch mutes the television.”
- “The subroutine ensures that the user is authorized.”
- “The magician’s assistant is for distracting the audience.”

We ascribe functions to biological stuff, artifacts, algorithms, personal roles. . .

How functions relate to means and ends

“That switch mutes the television.”

⇓

One can *use* the switch to mute the television.

⇓

Some *action* involving the switch will cause the television to be muted.

- Functions imply means-end relations.
- Doesn’t imply desirability of the end.
- Needed: means-end semantics
  - distinct of desirability
  - distinct from theory of practical reasoning

Initial analysis of means-end relations

- An end is some desirable condition – a *proposition*.
- A means is a way of making the end true.
- Means change things: means are *actions*.

Some controversies:

- Ends-in-themselves?
- Objects as means?

PDL syntax

Propositional Dynamic Logic is a logic of actions.

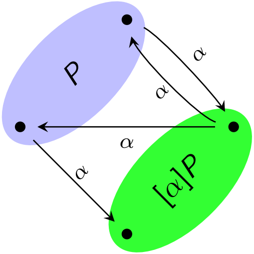
Basic types:

- a set **act** of *actions*,
  - Closed under:
    - *sequential composition* $\alpha;\beta$
    - *non-deterministic choice* $\alpha \cup \beta$
    - *test* $\varphi?$
    - *iteration* α^*
- a set **prop** of *propositions*.
  - Closed under:
    - boolean connectives,
    - dynamic operators $[\alpha]\varphi$, $\langle\alpha\rangle\varphi$.

Intuitions:

- $[\alpha]\varphi$: after doing α, φ *will* hold.
- $\langle\alpha\rangle\varphi$: after doing α, φ *might* hold.

PDL semantics



Possible world semantics with transition systems for each action α.

$w \xrightarrow{\alpha} w'$ means:
one can reach w' by doing α in w.

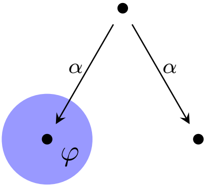
$$w \models [\alpha]\varphi \;\; \textit{iff} \;\; \forall\, w \xrightarrow{\alpha} w' \;.\; w' \models \varphi.$$

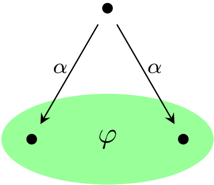
$$w \models \langle\alpha\rangle\varphi \;\; \textit{iff} \;\; \exists\, w \xrightarrow{\alpha} w' \;.\; w' \models \varphi.$$

Weak and strong means-end relations

A means is an action α that can realize one's end φ.

Two interpretations:





Weak: α *might* realize φ.

$w \models \langle\alpha\rangle\varphi$

Strong: α *will* realize φ.

$$w \models [\alpha]\varphi \land \underbrace{\langle\alpha\rangle\top}_{\alpha \text{ can be done.}}$$

Means distinguished by efficacy

Different means to a common end have different degrees of reliability.

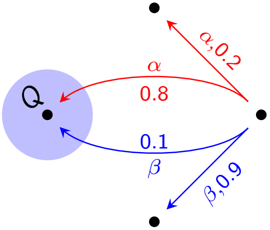
End: Get 12 points with one dart.

Three different means:

- Throw for 12.
- Throw for double 6.
- Throw for triple 4.

Efficacy: The degree of reliability of a means to an end.

From non-determinism to probabilities



Efficacy is a measure of likelihoods.

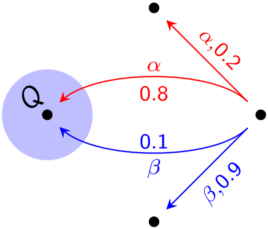
PDL includes non-determinism,
not probabilities.

<u>*Fix (semantic):*</u> use
probabilistic transition structures.

$w \xrightarrow[x]{\alpha} w'$ means that
doing α in w has probability x
of resulting in w'.

<u>*Write*</u>: $P(\, w \xrightarrow{\alpha} w' \,) = x$.

From non-determinism to probabilities



Syntactic fix?

- Probabilistic Computation Tree Logic (pCTL)?
  - Index dynamic operators, like $[\alpha]_{\geq x}$, $\langle\alpha\rangle_{\geq x}$.
  - Nesting requires picking x's.
- Probabilistic PDL?
  - Truth functional.
  - Assigns values in $[0,1]$ to world-formula pairs.
  - Logic in loose sense.
- Fuzzy PDL.

But probability $\neq$ fuzziness. . .

Slogan: Probabilities and fuzziness are different.

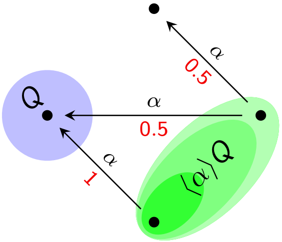
But one can use probabilities to define fuzzy predicates.

Hajek, et al., uses distributions on propositional formulas to define "Probably φ".

$$\text{Truth degree of "Probably } \varphi\text{"} = P(\varphi).$$

Reliability as a fuzzy proposition

"Reliably", like "Probably", is a vague operator.



In PDL:

$\langle\alpha\rangle\varphi \Leftrightarrow \alpha$ <u>will possibly</u> realize φ

In <u>fuzzy</u> PDL:

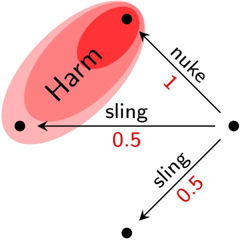
$\langle\alpha\rangle\varphi \Leftrightarrow \alpha$ <u>will *probably*</u> realize φ
$\Leftrightarrow \alpha$ <u>*reliably*</u> realizes φ

$$[\![\langle\alpha\rangle\varphi]\!](w) = \sum_{w'\in\mathcal{W}} P(w \xrightarrow{\alpha} w') \cdot [\![\varphi]\!](w').$$

- Like decision theory, we use means for expected outcomes.
- Unlike decision theory, there are no utilities involved.
- Elegant treatment of complex ends, like $\langle\alpha\rangle\varphi \wedge \langle\beta\rangle\psi$.

Fuzzy ends

An accidental advantage



Weapons are for causing harm.

Examples: slingshot, nuke

This end is fuzzy.

Fuzzy PDL allows for fuzzy ends.

A nuke is more effective in causing harm than a slingshot.

(Duh.)

Extending the logic to other connectives

Suppose J and L are cooperative but incommunicado.

J knows that L will either do

- m in order to realize P *or*
- n in order to realize Q.

He wants to ensure that L will succeed, whichever she chooses.

End: $\langle m\rangle P \wedge \langle n\rangle Q$.
Aim: maximize $\min\{[\![\langle m\rangle P]\!](w), [\![\langle n\rangle Q]\!](w)\}$.

$$[\![\varphi \wedge \psi]\!](w) = \min\{[\![\varphi]\!](w),\ [\![\psi]\!](w)\}$$

The semantics of fuzzy PDL

On formulas

$$
\begin{aligned}
[\![\langle\alpha\rangle\varphi]\!](w) &= \textstyle\sum_{w'\in\mathcal{W}} P(\ w \xrightarrow{\alpha} w'\ ) \cdot [\![\varphi]\!](w') \\
[\![\varphi \wedge \psi]\!](w) &= \min\{[\![\varphi]\!](w), [\![\psi]\!](w)\} && = [\![\varphi]\!] \cap [\![\psi]\!] \\
[\![\varphi \vee \psi]\!](w) &= \max\{[\![\varphi]\!](w), [\![\psi]\!](w)\} && = [\![\varphi]\!] \cup [\![\psi]\!] \\
[\![\neg\varphi]\!](w) &= 1 - [\![\varphi]\!](w) && = \mathcal{W} \setminus [\![\varphi]\!] \\
[\![\varphi \to \psi]\!](w) &= \begin{cases} 1 & \text{if } [\![\varphi]\!](w) \leq [\![\psi]\!](w), \\ [\![\psi]\!](w) & \text{else;} \end{cases} && = [\![\varphi]\!] \to [\![\psi]\!]
\end{aligned}
$$

The semantics of fuzzy PDL

On actions

$$[\![\alpha;\beta]\!](w)(w') \;=\; \sum_{w''\in\mathcal{W}} P(\,w \xrightarrow{\alpha} w''\,)\;\cdot\;P(\,w'' \xrightarrow{\beta} w'\,)$$

$$[\![\varphi?]\!](w)(w') \;=\; \begin{cases} [\![\varphi]\!](w) & \text{if } w = w'; \\ 0 & \text{else.} \end{cases}$$

$$\left.\begin{array}{r} [\![\varphi \cup \psi]\!](w)(w') \\ [\![\varphi^*]\!](w)(w') \end{array}\right\} \text{undefined.}$$

Logical properties

Validity and Soundness

Positive results:

- Axioms:
  - Usual axioms for this fuzzy logic (De Morgan and Implication axioms)
  - Composition: $[\alpha;\beta]\varphi \leftrightarrow [\alpha][\beta]\varphi$
- Rules:
  - Modus ponens, cut
  - Necessitation: $\varphi/[\alpha]\varphi$

Negative results:

- Axioms:
  - K: $[\alpha](\varphi \rightarrow \psi) \rightarrow ([\alpha]\varphi \rightarrow [\psi])$
  - Distributivity: $[\alpha](\varphi \wedge \psi) \leftrightarrow ([\alpha]\varphi \wedge [\alpha]\psi)$
  - Test: $[\psi?]\varphi \leftrightarrow (\psi \rightarrow \varphi)$

Logical properties

Completeness

I wish.

But not with these semantics.

Ongoing work. . .

Concluding remarks

- Include non-deterministic features (in paper).
- Add to formalization of functions (SPT 2005).
- Investigate better behaved semantics.

Adding efficacy to PDL

Concerns:

- Primary: Adding probabilities to transitions.
- Secondary: Fuzzy ends (like "causing harm").

Aims:

- Keep PDL as language for means-end relations.
- Minimal semantic changes.
- Truth-functional semantics.
- Include complex ends like $\langle\alpha\rangle\varphi \wedge \langle\beta\rangle\psi$.

Proposal: Interpret PDL as fuzzy logic.